Can't Lend or Won't Lend? Beliefs and the Macroeconomics of Financial Crises[‡]

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Abstract

Why do financial crises cause such deep recessions, and what can policy do about them? We distinguish between financial intermediaries' capacity to lend and their willingness to lend in a general-equilibrium model with heterogeneous intermediaries and occasionally binding leverage constraints. Two moments determine whether forward-looking intermediaries with room to lend offset constrained intermediaries' retrenchment and dampen downturns, or cut lending and amplify them: how strongly lending responds to expected returns and how those expectations respond to aggregate shocks. We estimate both using 40 million bank forecasts of borrower default probabilities. First, comparing banks assessing the same borrower at the same time, we show that a 1 p.p. increase in expected portfolio returns raises lending by 12%. Second, expectations overreact to aggregate shocks, leaving banks excessively pessimistic following adverse news. These two moments allow the calibrated model to generate dynamics for aggregate output during crises similar to those of a model driven by balance-sheet constraints alone, but with sharply different policy implications. In a crisis, the marginal intermediary is unconstrained and cuts lending because it expects low returns. Recapitalizations provide limited stimulus, whereas loan guarantees raise expected returns directly and stimulate output four times as much at the same fiscal cost. Our estimates also explain how booms build fragility: optimism raises leverage in booms, so a larger expansion deepens the subsequent recession.

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1 Introduction

Financial crises give rise to unusually deep recessions, with cumulative output losses as large as 15% of GDP.[1] Understanding why, and what policy can do about it, is a long-standing question in macroeconomics. One influential view emphasizes financial intermediaries' balance sheets: losses erode net worth, financial constraints force intermediaries to cut lending, and the contraction in credit depresses investment and output (Kiyotaki and Moore, 1997, Gertler and Kiyotaki, 2010, He and Krishnamurthy, 2013, Brunnermeier and Sannikov, 2014). This view points to recapitalizations of the financial sector as a powerful tool for restoring economic activity and provides a rationale for the considerable recapitalizations undertaken by governments during recent crises.[2]

Yet, as Stiglitz (2009) observed during the Global Financial Crisis, "*there is a difference between the ability to lend and the willingness to lend*." A second prominent view, dating back to Minsky (1977) and Kindleberger (1978), focuses on this willingness to lend by emphasizing the role of expectations: optimism fuels credit booms, while a shift to pessimism triggers deep crises (Bordalo et al., 2018, Maxted, 2024).

The first contribution of this paper is to develop a theoretical model that captures both views. In the model, financial intermediaries differ in their net worth and face occasionally binding leverage constraints. Some intermediaries *cannot* lend, since their constraint binds and their lending is pinned down by their balance sheets. Others are unconstrained and have room to lend but may be *unwilling* to do so, as their lending is governed by expected returns. The model shows that whether unconstrained intermediaries amplify or dampen crises is theoretically ambiguous and depends on two empirical moments: how strongly lending responds to expected returns and how expectations respond to aggregate shocks.

Our second contribution is to estimate these moments. Using 40 million forecasts of borrower default probabilities from the ECB's administrative credit registry, we compare forecast revisions across banks assessing the same borrower at the same time to isolate bank-level changes in beliefs and identify their effect on lending. A one-percentage-point increase in a bank's expected portfolio return raises its total lending by 12% after five quarters. We then examine how banks' forecast errors respond to identified aggregate shocks. We find that banks' expectations overreact: following adverse shocks, banks become excessively pessimistic, systematically predicting more defaults than subsequently materialize.

The final contribution of the paper is to show that these estimates lead to different conclu-

[1] See Hoggarth et al. (2002), Laeven and Valencia (2026).
[2] See, for example, Laeven and Valencia (2026).

sions about the effectiveness of policy during crises and the buildup of fragility before them. Our main result is that, once matched to our empirical moments, the model generates substantial amplification and aggregate dynamics similar to those of the standard financial accelerator, yet has very different policy implications. Recapitalizations expand lending by constrained intermediaries, but this additional lending lowers expected returns, and pessimistic unconstrained intermediaries cut back, largely offsetting the stimulus. Loan guarantees instead raise expected returns directly and stimulate investment and output more strongly at the same expected fiscal cost. Substantial amplification therefore need not imply that recapitalizations are a powerful stabilization tool. Matching the microdata also changes the model's predictions for the buildup of fragility before crises: optimism during booms raises leverage, so that larger booms are followed by deeper recessions, formalizing a view going back to Minsky (1977) and Kindleberger (1978) and helping rationalize the evidence in Jordà et al. (2013) and Mian et al. (2017).

We now describe each of these contributions in more detail.

A model of heterogeneous financial intermediaries. To capture the role of both balance sheets and expectations in lending decisions, we build a model with a continuum of financial intermediaries subject to a leverage constraint à la Gertler and Karadi (2011) and exposed to idiosyncratic shocks to their return on capital. These ingredients give rise to an endogenous distribution of net worth across intermediaries, so that lending is governed by balance sheets for some and by expectations for others. At the bottom of the distribution, intermediaries are up against their leverage constraint, so their lending is highly responsive to net worth but unresponsive to their own expectations of future returns. As net worth increases, intermediaries grow out of the constraint and their lending responding strongly to expected returns but little to net worth. The relative importance of balance sheets and expectations for aggregate outcomes therefore depends on the endogenous distribution of net worth.

In this setting, balance sheets and expectations interact in general equilibrium through the price of capital. Following an adverse aggregate shock, the price of capital falls, eroding net worth and forcing constrained intermediaries to cut lending. Under full-information rational expectations (FIRE), unconstrained intermediaries dampen the contraction, as they anticipate the recovery in capital prices and cut lending by less. When expectations overreact, unconstrained intermediaries instead become excessively pessimistic about the recovery and cut lending sharply. The two channels then reinforce each other. Because financial constraints make the price of capital more sensitive to the shock, pessimistic intermediaries expect a larger fall in future returns and cut lending by more, amplifying the contraction.

Whether unconstrained intermediaries amplify or dampen crises therefore depends on two moments. The first is how strongly lending responds to expected returns, which governs the relative importance of constrained and unconstrained intermediaries. The second is how expectations respond to aggregate shocks, which determines how expected returns move in a downturn.

Empirical evidence on lending and expectations. We turn to the data to measure these two moments. We first ask how forward-looking financial intermediaries are by estimating how strongly their lending responds to expected returns. Estimating this moment poses two challenges: (i) we need data linking intermediaries' expectations of returns to their lending decisions; and (ii) we need to isolate variation in these expectations that is uncorrelated with their balance sheets, since the economic conditions that shape expectations also affect intermediaries' capacity to lend. To address the first challenge, we introduce a new measure of banks' expectations, drawing on 40 million bank-level forecasts of borrower default probabilities from the European Central Bank's administrative credit registry. To address the second challenge, we extend the within-borrower identification of Khwaja and Mian (2008) and Amiti and Weinstein (2018) to banks' expectations. We compare how different banks revise their default forecasts for the same borrower at the same time. This allows us to separate a bank's own pessimism from its exposure to borrowers who are performing poorly, which would induce a comovement between its expectations and its capacity to lend. We translate these shifts in banks' beliefs into changes in expected portfolio returns using loan rates and recovery rates, and estimate how they affect total bank lending. Lending responds strongly to expected returns. A one-percentage-point increase in a bank's expected portfolio return raises its total lending by 12% after five quarters. These shifts in beliefs also propagate to the real economy, as firms whose lenders become more optimistic increase employment.

We next estimate the second of the two moments highlighted by the theory: how banks' expectations respond to aggregate shocks. Following Angeletos et al. (2021), we estimate the dynamic response of banks' forecast errors (the difference between realized and expected borrower defaults) to oil-supply news shocks (Känzig, 2021) and the main business-cycle shock (Angeletos et al., 2020). We find that expectations *overreact*: following an adverse shock, expected default probabilities rise by more than realized defaults, leaving banks overly pessimistic for several quarters.

Implications for policy during crises. The combination of expectation overreaction and financial constraints allows the model to match the forward-looking lending we document in the

microdata while generating substantial amplification, consistent with the aggregate evidence on financial crises. Both ingredients are crucial for this result, as the contraction is about a third smaller without financial constraints, and 40% smaller without overreaction. This allows our model, calibrated to the two empirical moments, to yield dynamics for aggregate investment and output similar to those of the standard financial accelerator. Yet matching the microdata leads to very different conclusions about the effectiveness of policy interventions.

In the standard financial accelerator, intermediaries cut lending because they *cannot* lend, so recapitalizations, which relax their constraints, strongly stimulate investment and output. In our model, the marginal intermediary is unconstrained and cuts credit because it is *unwilling* to lend. Recapitalizations still expand lending by constrained intermediaries, but the additional lending raises the price of capital and lowers expected returns, so pessimistic unconstrained intermediaries cut back. In general equilibrium the two forces largely offset, and the investment and output responses are only one-tenth and one-fifth as large as in the financial accelerator. Loan guarantees, under which the government covers part of the losses on new loans, instead raise expected returns directly, the margin to which unconstrained intermediaries are most responsive. As a result, they stimulate investment and output seven and four times as much as recapitalizations at the same fiscal cost.

Matching both models to the observed path of output, we then ask how the euro-area recession during the Global Financial Crisis would have unfolded without the bank recapitalizations undertaken at the time, which amounted to about 6% of GDP (Igan et al., 2019, Laeven and Valencia, 2026). In the financial accelerator, these recapitalizations prevented a much deeper recession, as without them the investment contraction would have been nearly twice as large. Through the lens of our model, instead, recapitalizations had a modest effect, reducing the investment and output contractions by 10% and 4%, respectively, as the marginal intermediary remained relatively unconstrained throughout the crisis. At the same cost for the government, loan guarantees would have been more effective, reducing the investment contraction by 8 percentage points and the output contraction by 1 percentage point.

Consistent with this mechanism, we also document empirically that larger recapitalizations are associated with stronger recoveries in European crisis episodes where banks report balance-sheet constraints as the main factor limiting their lending, but not in episodes where they mainly cite concerns about the economic outlook.

Implications for fragility before crises. Matching the microdata also leads to different predictions for the buildup of fragility before crises. Under FIRE, booms strengthen intermediaries' balance sheets, and larger expansions are followed by milder investment contractions.

In our model, by contrast, overreaction makes intermediaries excessively optimistic during the boom and leads them to increase leverage. When an adverse shock hits, higher leverage forces constrained intermediaries to cut lending more sharply. Because unconstrained intermediaries anticipate that a highly leveraged economy responds more strongly to bad news and revise their expected returns down accordingly, they retrench as well.

The same combination of overreaction and financial constraints that amplifies the bust thus also builds fragility in the boom, so that the larger the preceding expansion, the deeper the recession caused by the same adverse shock. This allows our model to formalize the idea that booms sow the seeds of the bust, dating back to Minsky (1977) and Kindleberger (1978), and to help rationalize rationalize the empirical evidence linking larger pre-crisis booms to more severe subsequent recessions (Jordà et al., 2013, Mian et al., 2017).

Related literature. We contribute to three strands of literature.

First, is the literature studying how the financial sector can turn adverse economic shocks into deep recessions (Kiyotaki and Moore, 1997, Bernanke et al., 1999, Gertler and Kiyotaki, 2010, He and Krishnamurthy, 2013, 2019, Brunnermeier and Sannikov, 2014, Bocola and Lorenzoni, 2023). Within this literature, four papers are closest to ours. Maxted (2024) and Krishnamurthy and Li (2025) study how beliefs and financial frictions jointly shape the buildup of financial crises and the subsequent downturns. Relative to them, we provide new estimates of banks' lending responses to expected returns in the microdata and use these estimates to discipline a model with heterogeneous intermediaries. In our framework, pessimistic lenders with spare capacity reinforce constrained banks' credit cuts, implying that large output and investment contractions to coexist with weak effects of recapitalizations. Bordalo et al. (2026) embed diagnostic expectations in a neoclassical model with heterogeneous firms. We share with them the result that larger booms are followed by deeper investment contractions. Our analysis differs in its focus on financial intermediaries, whose higher leverage makes the interaction between balance-sheet constraints and expectation overreaction central to our mechanism. This interaction gives rise to boom–bust amplification jointly for output, employment, and investment.[3] Bocola (2016) shows that sovereign default risk can induce financial intermediaries to deleverage for precautionary reasons, weakening the lending response to policies providing banks with subsidized funding. Our mechanism instead emphasizes intermediaries' unwill-

[3]Bordalo et al. (2018) embed diagnostic expectations in a frictionless model of credit and show that reversal of expectations can generate boom-bust dynamics. Our mechanism instead operates through financial frictions and the interaction between belief overreaction and intermediary leverage. Gertler et al. (2020a,b) show that expansions can make crises more likely, while we show that larger booms make the contraction following a given adverse shock more severe.

ingness to lend because of low expected returns, which we measure directly in banks' own forecasts, and yields new implications for recapitalization policy.[4]

Second, we add to a growing literature studying the role of subjective beliefs for macroeconomic dynamics. Closest to our work are Ma et al. (2022), Li and Ongena (2025), who estimate how borrower-level lending responds to banks' expectations of local economic conditions. Relative to them we make two contributions. First, we introduce a new measure of bank–borrower-level expectations. Second, our approach isolates shifts in banks' expectations about *portfolio-level* returns, allowing us to study the response of total lending rather than reallocations across borrowers, which alone do not distinguish constrained from unconstrained banks.[5] In doing so, we also contribute to an empirical literature that has been leveraging credit-registry data to identify credit-supply shocks (Khwaja and Mian, 2008, Amiti and Weinstein, 2018).[6] Our findings also complement existing evidence from information-provision experiments estimating how firms' behavior responds to their expectations (Coibion et al., 2018, 2020, Cuciniello and Paciello, 2026). Finally, our evidence that banks' expectations overreact to aggregate shocks also contributes to research documenting and interpreting departures from Full-information Rational Expectations (Bordalo et al., 2020, Angeletos et al., 2021, Afrouzi et al., 2023, Farroni and Tozzo, 2024, de Silva et al., 2025, Bonaglia et al., 2025).

Third, we relate to a growing body of work studying the macroeconomic implications of agent heterogeneity. While this literature has primarily focused on households (McKay and Reis, 2016, McKay et al., 2016, Kaplan et al., 2018, Auclert et al., 2024), a more recent strand has explored heterogeneity on the firm side (Ottonello and Winberry, 2020, Koby and Wolf, 2020, Jeenas, 2019, Winberry et al., 2025). We contribute to this line of work by studying heterogeneity across financial intermediaries and thus relate to recent papers on bank heterogeneity (Coimbra and Rey, 2024, Jamilov and Monacelli, 2026, Bellifemine et al., 2026).[7] Relative to these papers, we emphasize how the responsiveness of bank lending to balance sheets and expectations varies in the cross-section of intermediaries. We introduce new empirical moments

[4] Falato and Xiao (2024) show that the slow recovery of banks' beliefs after the 2008–09 crisis can account for the slow recovery of US bank lending. Relative to them, we (i) introduce a new quantitiative measure of banks' expectations, (ii) estimate the response of lending to expected returns, and (iii) characterize the interaction between banks' expectations and balance sheets via the price of capital in general equilibrium. Other related papers include Kozlowski et al. (2020), Fontanier (2025), Chiang and Zoch (2026), and Asriyan et al. (2026).

[5] Other related papers include Ma (2015), who documents that US banks with more optimistic CEOs expanded real estate lending by more in the run-up to the Global Financial Crisis, and D'Acunto et al. (2025) who use randomized information provision to shift Chinese loan officers' macroeconomic expectations and estimate the effect on their credit-supply decisions.

[6] A closely related application using firm trade data is di Giovanni et al. (2014).

[7] Other related papers include Abadi et al. (2023), Begenau et al. (2026), Abad et al. (2026).

to discipline the relative importance of these forces and show how they shape the effectiveness of crisis interventions.

Outline. The rest of the paper is organized as follows. Section 2 presents our heterogeneous-intermediary model and isolates the two moments that govern the transmission of shocks. Sections 3 and 4 introduce our data and estimate these moments, respectively the pass-through from banks' expectations of future returns to their lending and how these expectations themselves respond to aggregate shocks. Section 5 calibrates the model to these estimates and analyzes the implications for policy. Section 6 concludes.

2 Theory

In this section we present our heterogeneous-intermediary model, which nests both a balance-sheet channel and expectation channel. We show how the relative importance of these channels varies across the distribution of intermediary net worth: balance sheets matter more for the behavior of intermediaries closer to their leverage constraint, while expectations of future returns matter more for those further from it. We show that two moments govern whether unconstrained intermediaries' response to expectations offsets or reinforces the amplification generated by balance sheet constraints: how strongly lending responds to expected returns and how those expectations respond to aggregate shocks.

2.1 Model

Time $t = 0, 1, \ldots$ is discrete and runs forever. There is a continuum of financial intermediaries who face idiosyncratic shocks to the return on their capital holdings and are subject to a leverage constraint on their asset positions. These two ingredients give rise to a non-degenerate distribution of intermediaries' net worth.

Final-good producers. There is a unit mass of competitive final-good producers indexed by $f \in [0, 1]$. At the beginning of period t, firms observe aggregate productivity Z_t and choose capital k_{ft} and labor h_{ft}. They borrow resources $q_t k_{ft}$ from financial intermediaries, introduced momentarily, to finance the cost of installing capital, where q_t denotes the price of capital.

Firms are ex ante identical and face a common probability δ of failing, with failures independent across firms and time. They differ ex post only in their realized default outcomes. Failed firms produce no output, default on their loans with zero recovery, and have their capital fully

destroyed. Surviving firms produce output y_{ft} using a constant-returns-to-scale Cobb-Douglas technology:

$$y_{ft} = Z_t h_{ft}^{\alpha} k_{ft}^{1-\alpha}.$$

Upon survival, firms pay workers the wage bill of $w_t h_{ft}$ and transfer rental income $r_t^k k_{ft}$ and the resale value of capital $q_t k_{ft}$ to their lenders. As firm default is the sole source of capital depreciation, a fraction $1-\delta$ of aggregate installed capital survives each period.

Because firms face identical conditions when choosing inputs, we consider a symmetric allocation with $h_{ft} = H_t$ and $k_{ft} = K_t$, where H_t and K_t denote aggregate labor hired and capital installed before firm failures. Defining effective aggregate productivity as $A_t \equiv (1-\delta)Z_t$, aggregate output is:

$$Y_t = \int_0^1 y_{ft}\, df = (1-\delta) Z_t H_t^{\alpha} K_t^{1-\alpha} = A_t H_t^{\alpha} K_t^{1-\alpha}.$$

We assume that $\log Z_t$ (and hence $\log A_t$) follows an AR(1) process. Competitive factor markets imply that the wage $(1-\delta)w_t$ and rental rate on capital $(1-\delta)r_t^k$ averaged across surviving and failing firms equal the marginal products of the aggregate production function:

$$(1-\delta)w_t = \alpha A_t \left(\frac{K_t}{H_t}\right)^{1-\alpha}, \quad (1-\delta)r_t^k = (1-\alpha) A_t \left(\frac{H_t}{K_t}\right)^{\alpha}. \tag{1}$$

Capital good producer. Capital trades at price q_t. A continuum of competitive firms produce new capital by transforming the final good into investment goods, subject to adjustment costs $\Psi(I_t, K_t)$. Each capital good producer solves:

$$\max_{I_t} \; q_t I_t - I_t - \Psi(I_t, K_t),$$

which implies that the price of capital is pinned down by the marginal rate of transformation between the final good and the investment good:

$$q_t = 1 + \Psi_I(I_t, K_t), \tag{2}$$

where $\Psi_I(I_t, K_t)$ denotes the derivative of the adjustment cost function with respect to its first argument.

Financial intermediaries. There is a unit mass of financial intermediaries indexed by $b \in [0,1]$. Their liabilities consist of deposits b_{bt}, borrowed from households at the gross risk-free

rate $R_{t+1} \equiv 1 + r_{t+1}$, and net worth n_{bt}. Financial intermediaries use these funds to finance loans to final-good producers, k_{bt+1}, and pay dividends d_{bt}, yielding the familiar balance sheet identity:

$$q_t k_{bt+1} + d_{bt} = n_{bt} + b_{bt} + T_t^b,$$

where T_t^b is a lump-sum government transfer received each period.

Intermediaries face idiosyncratic risk in the return on their loan portfolios arising from differences in realized default rates. We define the realized default rate on intermediary b's portfolio as

$$\delta_{bt+1} = \delta + \varepsilon_{bt+1},$$

where ε_{bt+1} is a mean-zero, intermediary-specific default shock and can be thought of as capturing imperfect borrower diversification. As a result, the excess return intermediary b earns over the deposit rate, per euro lent, is given by

$$\mu_{bt+1} = \frac{(1 - \delta_{bt+1})(r_{t+1}^k + q_{t+1})}{q_t} - R_{t+1},$$

where $(1 - \delta_{bt+1})(r_{t+1}^k + q_{t+1})$ denotes the rental income net of default losses plus the resale value of surviving capital. The excess return inherits the randomness of ε_{bt+1} and is therefore stochastic at the intermediary level and we specify the distribution of default-rate shocks so that, conditional on aggregate variables $q_t \mu_{bt+1} \sim \mathcal{N}\left((1-\delta)(r_{t+1}^k + q_{t+1}) - q_t R_{t+1}, \sigma_\varepsilon^2\right)$.[8]

We allow intermediaries to differ in their expectations of default rates on their loan portfolio. Before choosing capital holdings and dividends, intermediary b draws a belief shock $\mathcal{S}_{bt} \sim \mathcal{N}(0, \sigma_{\mathcal{S}}^2)$, independent across intermediaries and time and independent of realized default shocks. The shock shifts intermediary b's expected default rate on its own portfolio from δ to $\delta - \mathcal{S}_{bt}$, leaving its expectations of the aggregate default rate unchanged. Positive realizations therefore make the intermediary more optimistic about future excess returns:

$$\widetilde{\mathbb{E}}_{bt} \mu_{bt+1} = \frac{(1 - \delta + \mathcal{S}_{bt}) \left(r_{t+1}^k + q_{t+1}\right)}{q_t} - R_{t+1}. \tag{3}$$

These belief shocks generate variation in expectations independent of fundamentals and can be interpreted as a reduced-form representation of differences in intermediaries' information or sentiment. They affect intermediaries' partial equilibrium lending behavior, while leaving the actual distribution of returns unchanged, allowing us to map the model to the empirical

[8] See Appendix A for the derivations.

estimates in Section 3.

Financial intermediaries derive utility $u(\cdot)$ from dividends and discount the future at rate β. There are two sources of exit: an exogenous exit probability $1-\varrho$, and endogenous exit upon insolvency, that is, whenever net worth turns negative. Upon exogenous exit, intermediaries consume their terminal net worth and derive flow utility $u(n)$. Insolvent intermediaries receive zero continuation value, and the government covers their shortfall through a deposit insurance scheme. Exiting intermediaries are replaced one-for-one by new entrants endowed with net worth n_e, keeping the mass of intermediaries constant.

Intermediaries are subject to two forms of financial frictions. The first is an equity issuance constraint $d \geq \underline{d}$. The second is a diversion constraint in the spirit of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011): each period, an intermediary can divert a fraction θ of its assets, abscond with them and retreat into autarky with access only to the risk-free bond, yielding value $V_t^\star(\theta q_t k')$.[9]

Let $V_t(n, \mathcal{S})$ denote the value of an intermediary entering period t with net worth n and a realization of the belief shock $\mathcal{S}$. The intermediary's problem can be expressed recursively as:

$$
\begin{aligned}
V_t(n,\mathcal{S}) = \max_{d\geq \underline{d},\, k'\geq 0} \; & u(d) + \beta \widetilde{\mathbb{E}}_t \left[\varrho V_{t+1}(n',\mathcal{S}') + (1-\varrho)u(n')\right] \\
\text{s.t.} \quad & n' = R_{t+1}(n - d + T_t^b) + q_t k' \mu_{bt+1}, \\
& V_t^\star(\theta q_t k') \leq \beta \mathbb{E}_t \left[\varrho V_{t+1}(n',0) + (1-\varrho)u(n')\right].
\end{aligned} \tag{4}
$$

Here $\widetilde{\mathbb{E}}_t$ denotes the intermediary's beliefs about future aggregate prices as well as its idiosyncratic belief shock. Given their beliefs intermediaries are internally rational, as in Adam and Marcet (2011). The inequality constraint captures the fact that, for depositors to be willing to supply funds, the value intermediaries get from diverting cannot exceed the continuation value. Depositors retain rational expectations, denoted by $\mathbb{E}_t$, so continuation payoffs in the constraint are evaluated under their beliefs.

The presence of idiosyncratic risk gives rise to a non-degenerate distribution of net worth. We denote by $g_t(n)$ its cross-sectional density at the start of period t, after exit and entry have occurred and before the current belief shocks are drawn. The intermediary's policy functions are $d_t(n,\mathcal{S})$ and $k_t'(n,\mathcal{S})$. Conditional on $\mathcal{S}$ and aggregate realizations, these policies and the actual return shock ε induce a Gaussian transition kernel $\mathbb{P}_t(n' \mid n, \mathcal{S})$. Integrating over belief shocks gives:

$$
\mathbb{P}_t(n' \mid n) = \int \mathbb{P}_t(n' \mid n, \mathcal{S})\, dF_{\mathcal{S}}(\mathcal{S}),
$$

[9] Appendix A.1 provides a detailed description of the problem under autarky.

where $F_{\mathcal{S}}$ denotes the distribution of belief shocks. The transition kernel $\mathbb{P}_t$ thus depends both on intermediaries' subjective beliefs through their policy functions, and on the actual distribution of returns. After accounting for exit and entry, the marginal distribution of net worth therefore evolves according to the Chapman–Kolmogorov equation:

$$\begin{aligned} g_{t+1}(n') = \varrho \int \mathbb{P}_t(n' \mid n)\, \mathbf{1}_{\{n' \geq 0\}} g_t(n)\, dn \\ + (1-\varrho)\delta_{n_e}(n') + \Xi_{t+1}\delta_{n_e}(n'), \end{aligned} \tag{5}$$

where the first term captures continuing intermediaries who survive the exogenous exit shock and remain solvent, the second term represents the inflow of new entrants replacing exogenous exits at the Dirac point mass $\delta_{n_e}(\cdot)$, and

$$\Xi_{t+1} = \varrho \int \mathbb{P}_t(n' < 0 \mid n)\, g_t(n)\, dn$$

denotes the mass exiting because of insolvency.

Household. There is a representative household deriving utility $v(C_t, H_t)$ from consumption C_t and hours worked H_t, and discounting the future at rate β^{HH}. It chooses consumption, hours, and deposits B_{t+1} to solve:

$$\begin{aligned} &\max_{\{C_t, B_{t+1}, H_t\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^{\text{HH}}\right)^t v(C_t, H_t) \\ &\text{s.t.} \quad B_{t+1} + C_t = (1-\delta) w_t H_t + R_t B_t - T_t + \Pi_t, \end{aligned} \tag{6}$$

where Π_t denotes profits rebated by capital good producers, and T_t are lump-sum taxes paid to the government.[10] Given this, the household's optimality conditions consist of an intratemporal labor-supply condition:

$$-\frac{v_h(C_t, H_t)}{v_c(C_t, H_t)} = (1-\delta) w_t, \tag{7}$$

and the standard consumption Euler equation:

$$\mathbb{E}_t \left(\frac{v_c(C_{t+1}, H_{t+1})}{v_c(C_t, H_t)} \right) \beta^{\text{HH}} R_{t+1} = 1. \tag{8}$$

Government. The government raises lump-sum taxes T_t from households to finance three expenditures: endowing newly born intermediaries with start-up capital n_e, insuring the deposits

[10] Idiosyncratic wage risk from failing firms is pooled across the household.

of insolvent intermediaries, and providing a per-period transfer T_t^b to the financial sector. The government runs a balanced budget, with no debt:

$$T_t = (1 - \varrho + \Xi_t)n^e + \mathcal{I}_t + T_t^b, \tag{9}$$

where $\mathcal{I}_t$ denotes the fiscal cost of deposit insurance, defined as:

$$\mathcal{I}_t = -\varrho \int \int n'_{t-1}(n,\varepsilon) \mathbb{1}_{n'_{t-1}(n,\varepsilon)<0} \, dG_{t-1}(n) dF_\varepsilon(\varepsilon),$$

where $n'_{t-1}(n,\varepsilon)$ denotes the law of motion of net worth and F_ε is the cumulative distribution function of ε.

Market clearing. There are five markets in the economy: one for final goods, one for capital goods, one for deposits, one for labor, and a rental market for capital.

Final-goods market clearing requires that production is split between household consumption, intermediaries' dividends, investment and adjustment costs paid by capital good producers:

$$Y_t = C_t + D_t + I_t + \Psi(I_t, K_t),$$

where $D_t = \int \int d_t(n,\mathcal{S}) dG_t(n) \, dF_\mathcal{S}(\mathcal{S})$ denotes aggregate dividends consumed by financial intermediaries.

Capital-goods market clearing implies the following law of motion for the aggregate capital stock:

$$I_t + (1-\delta)K_t = K_{t+1}, \quad \text{where} \quad K_{t+1} = \int \int k'_t(n,\mathcal{S}) dG_t(n) \, dF_\mathcal{S}(\mathcal{S}).$$

Deposit market clearing equates households' savings to aggregate intermediaries' borrowing,

$$B_{t+1} = \int \int b_t(n,\mathcal{S}) dG_t(n) \, dF_\mathcal{S}(\mathcal{S}).$$

Finally, market clearing in the labor market and rental market for capital trivially require that household labor supply equals demand for labor from final good producers and that the demand for capital from final good producers equals the existing capital stock.

Definition 1 (Full Information Rational Expectations Equilibrium)**.** *Given initial conditions for the capital stock K_0 and the cross-sectional distribution of intermediaries $G_0(n)$ and given an exogenous sequence for aggregate productivity $\{A_t\}$, a competitive Full Information Rational Expectations equilibrium is a sequence of prices $\{r_t^k, q_t, r_t, w_t\}$, quantities $\{C_t, I_t, T_t, K_t, H_t, B_{t+1}\}$, value functions $\{V_t(n,\mathcal{S})\}$, policy functions $\{d_t(n,\mathcal{S}), k'_t(n,\mathcal{S}), b_t(n,\mathcal{S})\}$, distributions $\{G_t(n)\}$ such that house-*

holds, final good producers, capital good producers and intermediaries optimize, i.e, equations (2), (4), (7) *and* (8) *are satisfied, the government budget constraint* (9) *holds, the distribution of intermediaries evolves according to* (5) *and the market for final goods, capital goods, bonds, labor and rental capital clear.*

We now proceed by defining the non-FIRE equilibrium, where we allow intermediaries to hold arbitrary expectations.

Definition 2 (Non-FIRE Equilibrium)**.** *Given initial aggregate capital K_0, the net-worth distribution $G_0(n)$, an actual productivity path $\{A_t\}_{t\geq 0}$, and intermediaries' subjective productivity forecasts $\{A^e_{t,s}\}_{s\geq t}$ at each date t, with $A^e_{t,t} = A_t$, a non-FIRE equilibrium consists of sequences of prices, aggregate quantities, intermediary value and policy functions, and net-worth distributions, together with intermediaries' subjective price forecasts, such that at each date t, intermediaries' forecasts of future prices coincide with those in a FIRE equilibrium starting from the current capital stock K_t and distribution $G_t(n)$ under their perceived productivity path $\{A^e_{t,s}\}_{s\geq t}$; intermediaries choose dividends, capital holdings, and borrowing optimally given current prices and their subjective forecasts, subject to their budget and incentive constraints; households and firms optimize given equilibrium prices, with households' expectations consistent with realized equilibrium sequences; the net-worth distribution evolves according to intermediaries' implemented policies, realized aggregate prices and returns, and idiosyncratic shocks; and all markets clear and the government budget constraint holds.*

This definition translates distorted beliefs about productivity into forecasts of endogenous prices through the model's FIRE equilibrium mapping. These forecasts need not coincide with realized prices: actual prices reflect intermediaries' choices under distorted beliefs. At each subsequent date, intermediaries form new forecasts using the updated aggregate state and net-worth distribution.

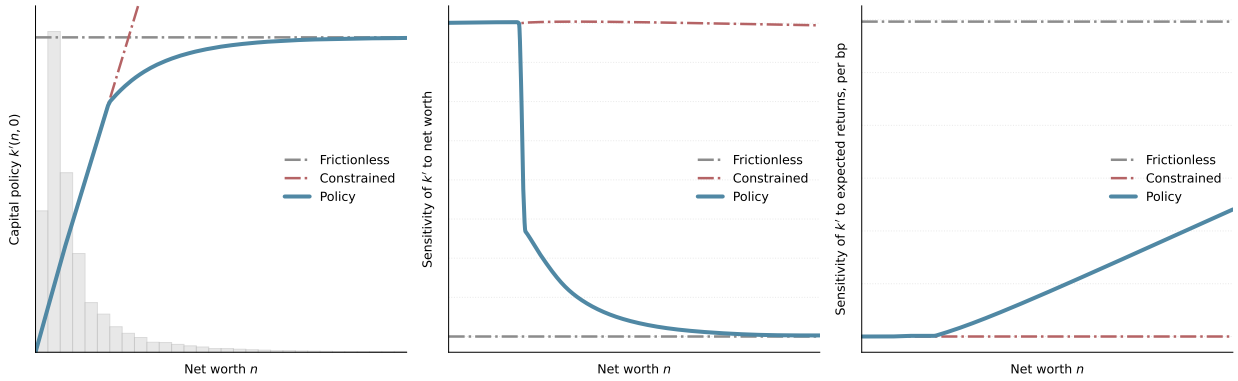
2.2 The Role of Balance Sheets and Expectations in Partial Equilibrium

We now examine how balance sheets and expectations shape intermediaries' policy functions. Throughout the rest of the paper, we consider the case in which $u(\cdot)$ takes the constant-absolute-risk-aversion (CARA) form:

$$u(d) = \frac{1 - e^{-\gamma d}}{\gamma}, \tag{10}$$

where $\gamma > 0$ is the coefficient of absolute risk aversion, governing financial intermediaries' risk bearing capacity. A key advantage of CARA utility is that, in the limit as net worth grows large,

Figure 1 – Expectations, balance sheets, and financial intermediaries' behavior



(a) Policy function and distribution

(b) Sensitivity of k' to net worth

(c) Sensitivity of k' to expected returns

Notes: panel (a) plots the stationary capital holdings policy function $k'(n,0)$ (solid line) and the stationary distribution of net worth $g(n)$ (gray bars). Panel (b) plots the sensitivity of capital holdings to net worth, defined as $\partial k'(n,0)/\partial n$. Panel (c) plots the partial-equilibrium response of capital holdings to expected default rates, $\partial k'(n,\mathcal{S})/\partial \mathcal{S}|_{\mathcal{S}=0}$, holding prices, aggregate beliefs, and the leverage constraint fixed. Across all three panels the dotted red line plots the behavior under a binding leverage constraint, while the dotted gray line shows the limit without financial frictions. See Table 1 in Section 5 for details on the calibration.

financial frictions cease to bind and the optimal portfolio choice becomes independent of net worth and is entirely pinned down by expected returns, thus allowing us to cleanly separate the two channels.[11]

Panel (a) of Figure 1 plots the stationary policy function for capital holdings evaluated at $\mathcal{S} = 0$, $k'(n,0)$, together with the stationary distribution of net worth $g(n)$. The policy function shows two limit cases, each isolating the role of one of the two forces present in our framework.

When net worth is low, the leverage constraint binds and capital holdings rise steeply with net worth: a marginal increase in n relaxes the constraint and allows the intermediary to expand its capital holdings. In this region, the model displays the standard financial accelerator mechanism, as capital demand is tied to current balance sheets.

At the other end of the distribution, for sufficiently high n, the constraint is slack and the policy flattens out, converging to the optimal frictionless portfolio choice (the Merton (1969) rule benchmark). In this regime, capital holdings are chosen on the basis of expected returns only, and a marginal increase in net worth has no effect on intermediaries' portfolio choice.

Between these limit cases there is an intermediate range of net worth for which the financial constraint no longer binds in a static sense, but its effects are still propagated because of

[11] Under our Gaussian assumption for idiosyncratic shocks ε_{bt+1}, expected CARA utility reduces to a mean-variance objective, which makes CARA a standard functional form in macroeconomic models of financial intermediation; see, e.g., Itskhoki and Mukhin (2021), Ray et al. (2024), and Gourinchas et al. (2025).

precautionary motives, so that optimal capital holdings are jointly pinned down by expected returns and current net worth.

Panels (b) and (c) of Figure 1 illustrate this distinction by showing how the sensitivity of capital holdings to net worth and expected future returns varies with n. More specifically, panel (b) plots the derivative $\frac{\partial k'(n,0)}{\partial n}$, characterizing the pass-through from net worth to lending. This pass-through is large at low n, where additional net worth relaxes the constraint. As n increases, however, it declines monotonically to zero, so that capital holdings become independent of net worth for sufficiently well-capitalized intermediaries. This pattern is consistent with the cross-sectional evidence on bank lending responses to equity documented in Bellifemine et al. (2026).

Panel (c) instead plots $\frac{\partial k'(n,\mathcal{S})}{\partial \mathcal{S}}\Big|_{\mathcal{S}=0}$, characterizing the partial equilibrium response of lending to greater optimism about default rates on the intermediary's own portfolio, holding current net worth, prices, aggregate beliefs, and the leverage constraint fixed.[12] As Equation (3) shows, $\mathcal{S}$ raises expected future returns. At low n, however, intermediaries cannot increase lending in response, since their capital holdings are pinned down by current net worth, so that $\frac{\partial k'(n,\mathcal{S})}{\partial \mathcal{S}}$ is exactly zero. As net worth increases and the constraint becomes slack, $\frac{\partial k'(n,\mathcal{S})}{\partial \mathcal{S}}$ turns positive and increases with n, as expected returns play a greater role in shaping intermediaries' lending.

Taken together, the three panels in Figure 1 show how the partial equilibrium response of lending to balance sheets and expectations varies in the cross-section of intermediaries. In our framework, the endogenous distribution of intermediaries' net worth therefore shapes the relative importance of these two forces for aggregate investment. Next, we turn to studying how these two channels interact for the propagation of shocks in general equilibrium.

2.3 Two-agent model: theoretical decompositions

To analytically isolate the role of financial intermediaries' expectations and balance sheets in the transmission of shocks, we consider a special case in which our model boils down to a simple two-agent framework, in the spirit of Campbell and Mankiw (1989), Galí et al. (2007), Bilbiie (2008). This allows us to exogenously vary the share of unconstrained intermediaries whose behavior is pinned down by expected returns.

[12] For given capital and dividend choices, the belief shock leaves the leverage constraint unchanged because continuation payoffs are evaluated under depositors' beliefs, which are unaffected by $\mathcal{S}$.

Constrained intermediaries. A share $1-\lambda$ of intermediaries are financially constrained, so that their choice of capital holdings k^c_{t+1} is proportional to their net worth n^c_t:

$$q_t k^c_{t+1} = \eta_t n^c_t. \tag{11}$$

These intermediaries correspond to the agents in the region responsive to net worth and unresponsive to expectations in Figure 1.

The behavior in (11) can be microfounded by taking the limit in which $\gamma \to 0$ and $\underline{d} \to 0$, so that utility is linear in dividends and the problem in (4) boils down to the original Gertler and Kiyotaki (2010), Gertler and Karadi (2011) framework, with $\eta_t = \frac{\mathbb{E}_t \beta\left[(1-\varrho)\theta^{-1}+\varrho\eta_{t+1}\right]R_{t+1}}{\mathbb{E}_t\left(1-\beta\left[(1-\varrho)\theta^{-1}+\varrho\eta_{t+1}\right]\mu_{t+1}\right)}$.[13]

Unconstrained intermediaries. The remaining share λ of intermediaries is financially unconstrained and their choice of capital holdings k^u_{t+1} is proportional to their expected risk-adjusted excess returns:

$$k^u_{t+1} = \nu_t \frac{\widetilde{\mathbb{E}}_t \mu_{t+1}}{\sigma^2_\varepsilon}. \tag{12}$$

These agents hence represent the intermediaries in Figure 1 that are unresponsive to net worth but responsive to expectations.

The behavior in (12) can be microfounded under the assumption that unconstrained intermediaries solve the same problem as in (4) with CARA utility and taking the limit in which financial frictions disappear, i.e., $\underline{d} \to -\infty, \theta \to 0$, which yields $\nu_t = q_t \left(\frac{1}{\gamma}\sum_{s=0}^{\infty} \widetilde{\mathbb{E}}_t \Pi_{j=1}^{s} R^{-1}_{t+1+j}\right)$. See Appendix A for more details.

Sequence space representation. To build intuition and isolate the moments that govern how expectations and balance sheets shape the transmission of shocks, in this section we linearize our model and cast it in the sequence space (Mankiw and Reis, 2007, Auclert et al., 2024). In the remainder of the paper, we solve the model fully nonlinearly.

Aggregate capital holdings of constrained intermediaries in period t can be expressed as a function of realized past and expected future paths of the rental rate on capital, capital prices, and the risk free rate:

$$K^c_{t+1} = \mathcal{K}^c\left(\{r^k_s, q_s, R_s\}_{s\leq t}, \{\mathbb{E}(r^k_s), \mathbb{E}(q_s), \mathbb{E}(R_s)\}_{s>t}\right). \tag{13}$$

[13] See Appendix A for derivations. As before $\widetilde{\mathbb{E}}_t$ captures the intermediaries own expectations about the future path of aggregate prices, while $\mathbb{E}_t$ captures depositors' beliefs.

As discussed above, constrained intermediaries are unresponsive to their expectations: they are up against their leverage constraint and do not intertemporally substitute. Expectations nonetheless appear in (13) because the tightness of the constraint (captured by the forward-looking leverage ratio η_t) is set by the intermediaries' depositors, in our framework households. The operator $\mathbb{E}(\cdot)$ thus represents households' rather than intermediaries' beliefs.

Let $\mathrm{d}x_t$ denote the first-order deviation of x_t from its steady-state value, and stack sequences as $\mathrm{d}\boldsymbol{x} \equiv (\mathrm{d}x_0, \mathrm{d}x_1, \ldots)'$. Linearizing (13) gives:

$$\mathrm{d}\boldsymbol{\mathcal{I}}^c = \boldsymbol{\Theta}^c_B \left(\mathrm{d}\boldsymbol{\mu} + \tfrac{1}{\bar{\eta}} \mathrm{d}\boldsymbol{R} \right) + \boldsymbol{\Theta}^c_F \left(\mathbb{E}\, \mathrm{d}\boldsymbol{\mu} + \tfrac{1}{\bar{\eta}}\, \mathbb{E}\, \mathrm{d}\boldsymbol{R} \right) - \bar{K}^c \boldsymbol{D}\, \mathrm{d}\boldsymbol{q}, \tag{14}$$

where $\mathcal{I}^c_t = k^c_{t+1} - (1-\delta)k^c_t$ is investment financed by constrained intermediaries, $\boldsymbol{D} \equiv \boldsymbol{I} - (1-\delta)\boldsymbol{L}$, $\boldsymbol{L}$ is the lag operator, and $\bar{\eta}$ denotes steady state leverage. The first term in (14) represents the balance sheet channel at play in the model. Past returns on intermediaries' equity feed into current balance sheets and hence shape their ability to lend. This channel is summarized by $\boldsymbol{\Theta}^c_B$, a lower triangular matrix capturing the backward-looking nature of constrained intermediaries' lending behavior. The second term instead captures the extent to which depositors' expectations about future returns move the tightness of the leverage constraint, as summarized by the forward-looking, upper triangular matrix $\boldsymbol{\Theta}^c_F$. Finally, the last term in (14) is a valuation effect coming from the fact that the leverage constraint puts a limit on the market value of capital holdings. A higher price of capital hence mechanically reduces the real capital that a given net worth can support.

By contrast, unconstrained intermediaries' capital holdings are independent of their net worth and hence of past returns. Their investment is instead pinned down solely by their own expectations of future returns:

$$K^u_{t+1} = \mathcal{K}^u \left(\left\{ \widetilde{\mathbb{E}}(r^k_{s+1}), \widetilde{\mathbb{E}}(q_s), \widetilde{\mathbb{E}}(R_{s+1}) \right\}_{s \geq t} \right) \tag{15}$$

Because unconstrained intermediaries face no leverage limit, the expectations entering (15) are no longer depositors' but the intermediaries' own, which we denote by $\widetilde{\mathbb{E}}(\cdot)$. Similarly to before, the linearized investment of unconstrained intermediaries can be expressed as:

$$\mathrm{d}\boldsymbol{\mathcal{I}}^u = \boldsymbol{\Theta}^u \widetilde{\mathbb{E}} \mathrm{d}\boldsymbol{\mu} + \bar{K}^u \boldsymbol{D}\, \mathrm{d}\boldsymbol{q} - \boldsymbol{B}\, \widetilde{\mathbb{E}} \mathrm{d}\boldsymbol{R}. \tag{16}$$

The first term in (16) represents the expectation channel at play in our setting, as it captures how intermediaries' expectations of future returns shape their investment. The matrix $\boldsymbol{\Theta}^u$ governs the partial equilibrium pass-through from expectations of future excess returns to investment

choices and is upper triangular, since unconstrained intermediaries' choices are independent of past returns. The second term converts excess returns in per-unit-of-capital terms, what ultimately matters for unconstrained intermediaries. Finally, the last term captures the effect of movements in the risk-free rate on intermediaries' desired capital holdings.[14]

Aggregation. We define aggregate investment financed by the financial sector as the weighted average of constrained and unconstrained intermediaries' lending and denote it by $\mathcal{I}_t$, so that:

$$\mathrm{d}\boldsymbol{\mathcal{I}} = (1-\lambda)\mathrm{d}\boldsymbol{\mathcal{I}}^c + \lambda \mathrm{d}\boldsymbol{\mathcal{I}}^u. \tag{17}$$

In this two-intermediary framework, λ governs the relative strength of the balance-sheet and expectation channels. A larger share of unconstrained intermediaries (i.e., a higher λ) makes aggregate investment more responsive to intermediaries' expectations, placing more weight on the upper-triangular $\boldsymbol{\Theta}^u$. Increasing the share of constrained intermediaries (i.e., decreasing λ) instead ties investment more tightly to past returns, as it strengthens the role of the balance sheet channel through the lower-triangular $\boldsymbol{\Theta}^c_B$.

The model also nests two familiar benchmarks as limiting cases. As $\lambda \to 0$, the model reduces to the canonical Gertler and Kiyotaki (2010), Gertler and Karadi (2011) economy, and only the balance-sheet channel is active. As $\lambda \to 1$, the economy boils down to the frictionless benchmark where all intermediaries are unconstrained and investment is pinned down by expected future returns only.

Investment response to aggregate shocks. To simplify notation, in what follows we make three assumptions. Appendix A relaxes all three and derives our results in the more general case. First, we make a small open economy assumption, so that the risk-free rate is constant and $\mathrm{d}\boldsymbol{R} = \mathbf{0}$. Second, we assume households have GHH preferences (Greenwood et al., 1988) and labor supply is perfectly elastic, so that changes in the rental rate on capital are independent of the capital stock and only depend on aggregate productivity. Finally, we make parametric restrictions to the investment adjustment cost function $\Psi(\cdot,\cdot)$ so that $\Psi_{II}(\bar{I},\bar{K}) = 1$ and $\Psi_{IK}(\bar{I},\bar{K}) = 0$.

Under these assumptions, the following proposition characterizes the general equilibrium investment response to a TFP shock when intermediaries from expectations under full information rational expectations, so that $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{X} = \mathbb{E}\,\mathrm{d}\boldsymbol{X} = \mathrm{d}\boldsymbol{X}$ for every variable X.

[14] $\boldsymbol{\Theta}^c_B$, $\boldsymbol{\Theta}^c_F$, $\boldsymbol{\Theta}^u$, and $\boldsymbol{B}$ are functions of parameters only. Full derivations are in Appendix A.

Proposition 1. *The first order response of investment to an aggregate productivity shock in a small open economy with perfectly elastic labor supply under Full Information Rational Expectations is given by:*

$$\mathrm{d}\boldsymbol{\mathcal{I}} = \underbrace{\boldsymbol{\Theta}(\lambda)\,\mathrm{d}\boldsymbol{A}}_{\textit{Direct effect}} + \underbrace{\Big(\boldsymbol{\Theta}(\lambda)((1-\delta)\boldsymbol{I} - \bar{R}^k\boldsymbol{L}) + \boldsymbol{\mathcal{R}}(\lambda)\Big)\,\mathrm{d}\boldsymbol{\mathcal{I}}}_{\textit{Indirect effect}} \tag{18}$$

where $\boldsymbol{\Theta}(\lambda) \equiv \lambda\boldsymbol{\Theta}^u + (1-\lambda)(\boldsymbol{\Theta}^c_B + \boldsymbol{\Theta}^c_F)$ *and* $\boldsymbol{\mathcal{R}}(\lambda) \equiv (\lambda\bar{K}^u - (1-\lambda)\bar{K}^c)\,\boldsymbol{D}$.

Proof. See Appendix A. □

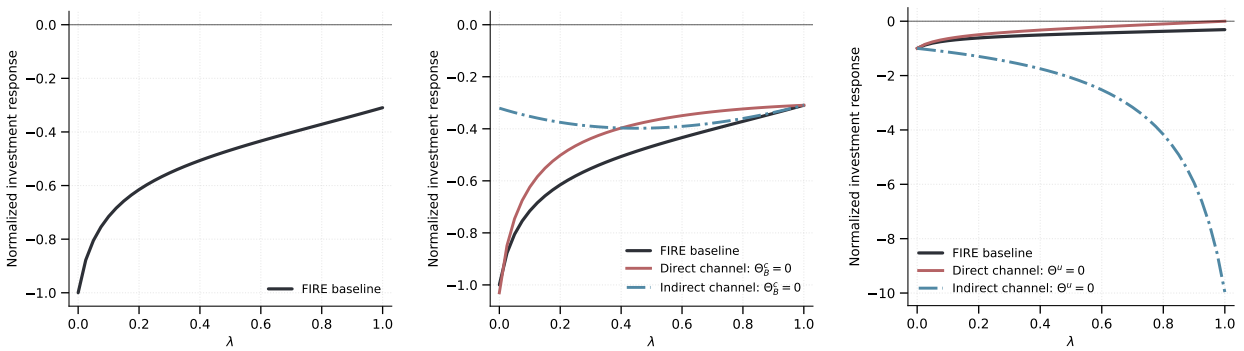
Balance sheets and expectations under FIRE: amplification and stabilization. Proposition 1 decomposes the general equilibrium investment response to an aggregate productivity shock into a direct and an indirect effect. The direct effect captures the response to the change in the rental rate of capital $\mathrm{d}\boldsymbol{r}^k$ induced by the shock, holding the price of capital fixed. The indirect effect captures the general equilibrium feedback between investment and the price of capital, as the initial investment response changes the path of q_t, generating capital gains or losses that feed back into investment.

Since rental rates and capital prices both enter the intermediaries' problem through their common effect on excess returns, the same matrix $\boldsymbol{\Theta}(\lambda)$ governs the direct and the indirect effect. This matrix combines three economic channels. The $\lambda\boldsymbol{\Theta}^u$ term captures the expectation channel, the investment response of unconstrained intermediaries to their expectations of future excess returns. The $(1-\lambda)\boldsymbol{\Theta}^c_B$ component is the balance sheet channel, reflecting the fact that constrained intermediaries' investment is shaped by their net worth, a backward-looking variable that depends on past returns. Finally, the $(1-\lambda)\boldsymbol{\Theta}^c_F$ term captures the effect of future returns on the tightness of constrained intermediaries' leverage constraint.[15]

Panel (a) of Figure 2 plots the on-impact investment response characterized in Proposition 1 as a function of the share of unconstrained intermediaries λ. Following a negative productivity shock, the price of capital drops on impact and then mean-reverts to steady state. The resulting erosion in net worth forces constrained intermediaries to cut lending. Under FIRE, unconstrained intermediaries anticipate future capital gains and cut investment by less than they otherwise would, partly offsetting this amplification. As a result, increasing λ dampens the response to the shock, with the contraction in the frictionless benchmark ($\lambda = 1$) less than half

[15] The indirect effect also features a revaluation term, $\boldsymbol{\mathcal{R}}(\lambda)$. For constrained intermediaries, this term arises because changes in the price of capital mechanically affect the left-hand side of the leverage constraint. For unconstrained intermediaries, the term simply rescales excess returns from per-dollar to per-unit-of-capital units.

Figure 2 – Unconstrained intermediaries dampen the response to shocks



(a) On-impact investment response

(b) Decomposition: balance sheets

(c) Decomposition: expectations

Notes: panel (a) plots the on-impact investment response to an aggregate productivity shock as a function of the share of unconstrained intermediaries λ, as characterized by Proposition 1. We normalize the shock so that the response for $\lambda = 0$ is -1%. Panel (b) decomposes the role of balance sheets, by recomputing the counterfactual response from (18) shutting down the $\mathbf{\Theta}_B^c$ Jacobian in the direct channel only (red line) and in the indirect channel only (blue line). Panel (c) does the same for the $\mathbf{\Theta}^u$ Jacobian.

as large as in the the fully constrained limit ($\lambda = 0$).[16] Even a small share of unconstrained intermediaries is sufficient to offset most of this amplification, as the stabilizing effect is strongest at low λ.

The indirect effect operating through the general equilibrium response of the price of capital is central to both the amplification generated by constrained intermediaries and the stabilization provided by unconstrained intermediaries. To show this, panels (b) and (c) of Figure 2 report counterfactual responses obtained from Proposition 1 by shutting down the balance sheet (setting $\mathbf{\Theta}_B^c = \mathbf{0}$) and expectation (setting $\mathbf{\Theta}^u = \mathbf{0}$) channels, respectively, in either the direct or the indirect effect. Panel (b) shows how the amplification associated with the balance sheet channel remains largely intact when this channel is shut down in the direct effect (red) but vanishes when it is shut down in the indirect effect (blue). This is because the drop in the price of capital erodes intermediaries' balance sheets, forcing them to cut investment. Similarly, panel (c) shows that the stabilization associated with unconstrained intermediaries fully operates through the indirect effect and intermediaries' expectations of the future price of capital.

Taken together, Proposition 1 and Figure 2 show that unconstrained intermediaries play a strong stabilizing role under FIRE, dampening the economy's response to shocks. Expectations of future capital prices are central to this result, as unconstrained intermediaries anticipate the

[16] In Appendix A, we prove that there exists a threshold $\rho^\star$ for the persistence of the aggregate productivity shock such that increasing λ dampens the impact investment response whenever shock persistence is below $\rho^\star$.

recovery in q_t thus cutting lending by less.

Stabilization and amplification when expectations overreact. As the previous results highlight the importance of expectations of future capital prices, we now examine how departures from FIRE affect the role of unconstrained intermediaries in the transmission of aggregate shocks.

To do so, we consider a simple expectation formation rule in which departures from FIRE are parametrized by a scalar χ, with $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} = \chi \mathrm{d}\boldsymbol{A}$.[17] Under this formulation, $\chi = 1$ recovers the FIRE benchmark, while $\chi > 1$ ($\chi < 1$) corresponds to expectation *overreaction* (underreaction): following an aggregate shock, expectations move by more (less) than realized outcomes.[18]

We also assume that, given their expectations of aggregate productivity, intermediaries form expectations of future returns using the model-consistent mapping from exogenous productivity to endogenous aggregate outcomes. Rearranging equation (18) in Proposition 1, we denote the mapping to capital prices by $\boldsymbol{\mathcal{J}}(\lambda) \equiv \left[\boldsymbol{I} - \left(\boldsymbol{\Theta}(\lambda)((1-\delta)\boldsymbol{I} - \bar{R}^k\boldsymbol{L}) + \boldsymbol{\mathcal{R}}(\lambda)\right)\right]^{-1}\boldsymbol{\Theta}(\lambda)$. The following Proposition characterizes how the stabilizing role of unconstrained intermediaries changes when expectations depart from FIRE.

Proposition 2. *The general equilibrium investment response to an aggregate productivity shock* $\mathrm{d}\boldsymbol{A}$ *given an arbitrary path for intermediaries' expectations* $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}$ *is:*

$$\mathrm{d}\boldsymbol{\mathcal{I}} = \boldsymbol{\mathcal{J}}(\lambda)\mathrm{d}\boldsymbol{A} + \widetilde{\boldsymbol{\mathcal{J}}}(\lambda)\left(\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} - \mathrm{d}\boldsymbol{A}\right), \tag{19}$$

where $\widetilde{\boldsymbol{\mathcal{J}}}(\lambda) \equiv \lambda\boldsymbol{\mathcal{M}}(\lambda)\left(\boldsymbol{\Theta}^u + \boldsymbol{\Theta}^u(1-\delta)\boldsymbol{\mathcal{J}}(\lambda)\right)$. *When the expectation formation process of intermediaries follows* $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} = \chi\mathrm{d}\boldsymbol{A}$, (19) *above becomes:*

$$\mathrm{d}\boldsymbol{\mathcal{I}} = \boldsymbol{\mathcal{J}}(\lambda)\mathrm{d}\boldsymbol{A} + (\chi - 1)\,\widetilde{\boldsymbol{\mathcal{J}}}(\lambda)\mathrm{d}\boldsymbol{A}. \tag{20}$$

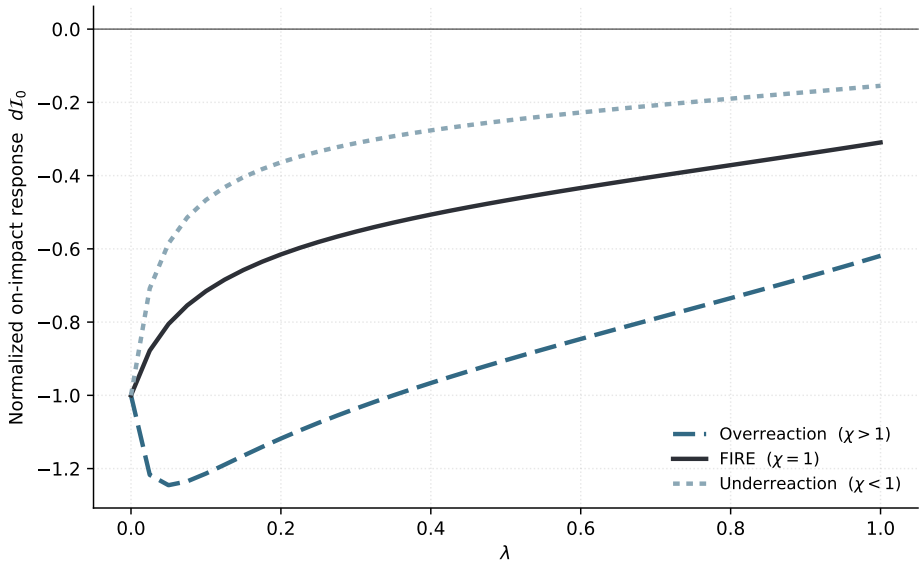
Proof. See Appendix A. □

Proposition 2 characterizes the equilibrium investment response to an aggregate productivity shock when expectations depart from FIRE. For an arbitrary expected path $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}$, Equation (19) decomposes this response into the FIRE response characterized in Proposition 1 and the

[17] Throughout the paper, we maintain that depositors (households) form expectations under full information rational expectations.

[18] Appendix A rederives our results in the more general case where intermediaries update expectations of future aggregate productivity as time passes by, i.e., the case in which χ is a linear operator rather than a scalar.

Figure 3 – Expectations, balance sheets, and the transmission of aggregate shocks



Notes: on-impact investment response to an aggregate productivity shock as a function of the steady-state share of unconstrained intermediaries λ as characterized in Proposition 2. The black solid line is the case in which expectations are formed according to FIRE, while the dotted light blue and dashed dark blue lines consider the case where expectations underreact and overreact, respectively.

response to a pure shock to intermediaries' beliefs, captured by the forecast error $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} - \mathrm{d}\boldsymbol{A}$. Equation (20) shows that, under our expectation formation process, this forecast error term can be expressed as $(\chi - 1)\mathrm{d}\boldsymbol{A}$.

Figure 3 plots the on-impact investment response characterized in Proposition 2 as a function of the share of unconstrained intermediaries λ, for three values of χ: the FIRE benchmark ($\chi = 1$, solid black line), overreaction ($\chi > 1$, dashed dark-blue line) and underreaction ($\chi < 1$, dotted light-blue line). Overreaction of expectations weakens the stabilizing role of unconstrained intermediaries. Following a negative shock, unconstrained intermediaries cut lending more sharply than under FIRE, as they expect future productivity to be lower than subsequently realized and therefore forecast lower future capital prices. For sufficiently strong overreaction, unconstrained intermediaries can even *amplify* rather than dampen the response to shocks.[19] The same mechanism implies that underreaction of expectations strengthens the stabilizing role of unconstrained intermediaries relative to the FIRE benchmark.

Moreover, Figure 3 shows that the amplifying effect of overreaction can be hump-shaped in λ, even though only unconstrained intermediaries respond to expectations in partial equilibrium. The reason for this is that when expectations overreact, there is a general equilibrium

[19] In Appendix A we prove analytically both that $\chi > 1$ reduces the stabilizing role of unconstrained intermediaries and that there exists a threshold $\chi^\star$ such that unconstrained intermediaries amplify the investment response when $\chi \geq \chi^\star$.

interaction between constrained and unconstrained intermediaries operating through the price of capital. A higher λ weakens the balance sheet amplification of capital prices characterized in Proposition 1 and captured by the $(1-\delta)\mathcal{J}(\lambda)$ term in $\widetilde{\mathcal{J}}(\lambda)$. As unconstrained intermediaries account for this weaker response when forming expectations, the same pessimism about aggregate productivity translates into a smaller decline in expected future capital prices. Therefore, a larger λ weakens the effect of a given degree of overreaction on expected excess returns, while increasing the mass of intermediaries whose lending responds to these expectations.

Taking stock. Unconstrained intermediaries can therefore either offset constrained lenders' retrenchment and dampen downturns, or cut lending and amplify them. Our model highlights the two moments that shape this interaction between expectations and balance sheet constraints. The first is the *partial equilibrium* pass-through from intermediaries' expectations of future returns to their lending choices, captured by the average lending sensitivity $\frac{\partial k'(n,\mathcal{S})}{\partial \mathcal{S}}$ in Figure 1 and by $\lambda\boldsymbol{\Theta}^u$ in the simple two-agent model. As Proposition 2 makes clear, another moment is needed to map this partial equilibrium object into its general equilibrium implications: how these expectations themselves respond to aggregate shocks, governed by χ.

In the next two sections we turn to the data to estimate these moments.

3 How Bank Lending Responds to Expectations

To understand the role of financial intermediaries' beliefs in aggregate fluctuations, we first ask how forward-looking their lending decisions are. Answering this question requires estimating the first of the two moments highlighted by our theory: how much banks cut lending when they expect lower returns.

Estimating this moment poses two challenges. First, it requires data linking intermediaries' expectations of future returns to their lending decisions. Second, it requires isolating variation in these expectations that is uncorrelated with intermediaries' balance-sheet conditions. This is difficult, as intermediaries' expectations respond to economic conditions that also affect their balance sheets and capacity to lend. A growing literature overcomes this identification problem using information treatments embedded in surveys, which provide exogenous variation in respondents' beliefs (Coibion et al., 2020, Coibion and Gorodnichenko, 2026). While fruitful for households and firms, this approach is less well suited when it comes to sophisticated financial intermediaries, for whom there may be less scope to shift beliefs through information

provision.[20]

We address the first challenge by introducing a new measure of banks' beliefs about individual borrowers from administrative credit-registry data. To address the second challenge, we exploit the panel structure of these data and use within-borrower, between-bank differences in beliefs to isolate variation in banks' expectations from other determinants of lending.

In the remainder of this section, we describe the data, present our identification strategy, and present the results.

3.1 Data: Measuring Bank Expectations

AnaCredit. Our main data source is the Analytical Credit Dataset (henceforth, AnaCredit), an administrative credit registry maintained by the European Central Bank. AnaCredit provides monthly loan-level data on the quasi-universe of business loans extended by credit institutions domiciled in the euro area, beginning in September 2018. Reporting is mandatory for exposures with a total borrower-level commitment above €25,000, and covers both standard term loans and credit lines. The data contain detailed information on each credit instrument, including lender and borrower identifiers, outstanding and committed undrawn amounts, collateral, and default status. Throughout our analysis, we aggregate the data at quarterly frequency and restrict the sample to lenders located in Germany, France, Italy, or Spain, the four largest economies in the Euro area. Our final dataset consists of over 34 million observations covering 29 quarters, 3,304,743 unique bank-firm relationships, 1,123,197 unique firms and 177 unique banks. Summary statistics are reported in Table B.1 in Appendix B.

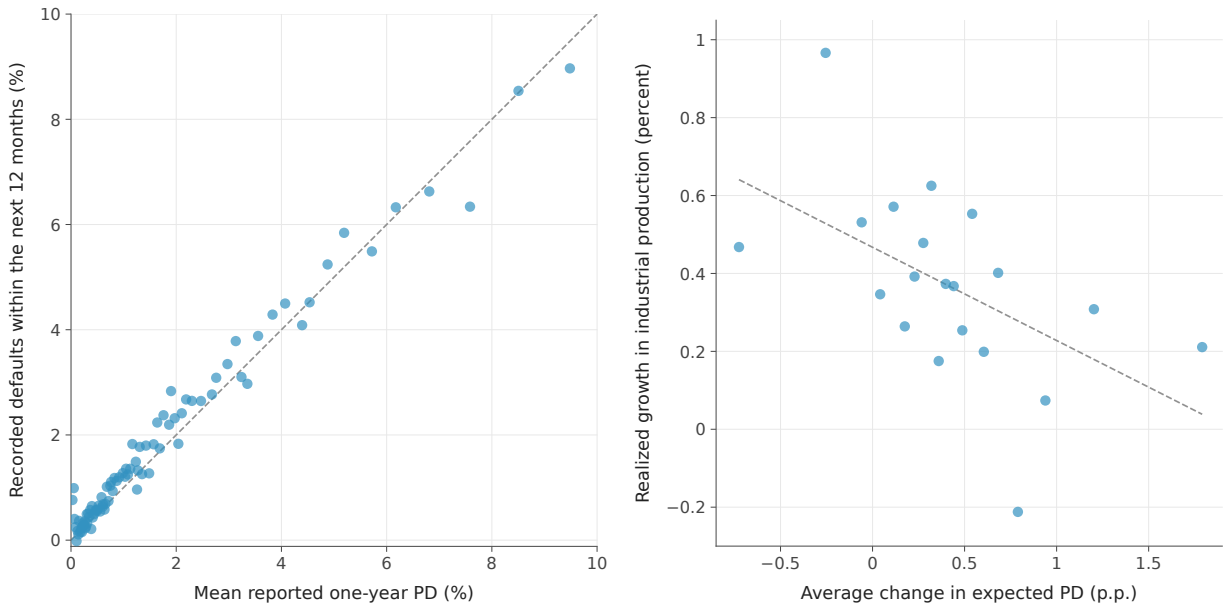
The European credit market is particularly well suited to our analysis for two reasons. First, bank lending is the primary source of corporate debt financing in the region, accounting for roughly 60% of total firm debt in 2024, compared with 20% for market-based finance.[21] Second, given the importance of bank financing, firms typically maintain relationships with several lenders. In our sample, more than 70% of SMEs and large firms borrow from multiple banks at the same time, and these firms account for more than 80% of outstanding credit.[22] This is important for our empirical approach, as we exploit differences in beliefs across banks lending

[20] A notable exception is D'Acunto et al. (2025), who use randomized information provision to shift Chinese loan officers' macroeconomic expectations and estimate the effect on their credit-supply decisions. Gorodnichenko and Yin (2024) and Beutel and Weber (2025) use information treatments with retail investors to study how expectations affect their portfolio choice.

[21] See Pelizzon et al. (2025), European Systemic Risk Board (2025). These figures exclude intra-group financing and trade credit.

[22] Sette and Gobbi (2015) report that 90% of firms in their Italian sample borrow from more than one bank. The corresponding shares are 87% in the German sample of Ongena et al. (2012) and 72% in the Spanish sample of Karaivanov et al. (2010).

Figure 4 – Reported PDs and realized outcomes



(a) Realized vs expected defaults **(b)** Expected PD and macroeconomic outlook

Notes: panel (a) is a binned scatter plot the mean reported PD (x-axis) against the share of observations for which the borrower defaults within the next 12 months (y-axis). The dashed line is the 45-degree line, corresponding to the case in which reported PDs equal realized default rates. Panel (b) is a binned scatter plot of the average change in expected probability of default reported by banks in our sample for borrowers in a given sector and country (x-axis) against realized growth in the industrial production index (or service production index, for non-manufacturing sectors) of that sector-country 1-year ahead.

to the same firm at the same point in time.

A unique feature of our dataset is that it provides a direct measure of lenders' beliefs about individual borrowers. For each borrower, banks are required to report their forecast of the probability of default over the following twelve months. As this measure is central to our analysis, we describe it in detail next.

Expectations. Our main variable of interest is banks' forecasts of each borrower's probability of default over the following twelve months (henceforth, PD). Because of the existence of cross-default clauses, banks are required to forecast defaults at the *borrower level* rather than the loan level.[23]

Banks produce these probabilities internally for regulatory purposes, under the Internal Ratings-Based approach (henceforth, IRB) introduced by the Basel II agreement of 2007. Under IRB, each bank rates the riskiness of its borrowers using proprietary statistical models, and the

[23]Under Article 178 of the EU Capital Requirements Regulation, a borrower is considered in default if either (i) the lender judges that the borrower is unlikely to repay its credit obligations (unlikeliness-to-pay criterion); or (ii) the borrower is more than 90 days past due on a credit obligation (days-past-due criterion).

resulting PDs serve as direct inputs into their capital requirements. Both the initial models and any subsequent relevant modifications require supervisory approval by the ECB, which conducts back-testing, audits, and on-site investigations to verify compliance. Since its inception, the IRB approach has been widely adopted, particularly by larger institutions: as of 2024, IRB lending accounted for more than 70% of total loan volume to non financial corporations in the euro area (European Banking Authority, 2025).

Reported default probabilities track subsequent default rates closely, consistent with the regulatory scrutiny of banks' forecasting performance. Panel (a) of Figure 4 groups bank–firm–quarter observations into bins of reported PD and plots each bin's realized one-year default rate against its average reported PD. Borrowers assigned higher default probabilities subsequently default more often, with the bin averages broadly following the 45-degree line. This pattern suggests reported PDs contain meaningful information about borrowers' future default risk.

When forming these forecasts, banks also appear to take the macroeconomic outlook into account. Panel (b) of Figure 4 shows a binned scatterplot of realized one-year-ahead industrial (or service) production growth against average PD revisions at the sector-by-country-by-quarter level. Sectors for which banks revise default probabilities upward subsequently experience significantly lower growth, suggesting that banks take broader economic conditions into account when assessing borrower risk. This is consistent with the regulatory requirements that PD estimates reflect both current and foreseeable economic conditions and that model assumptions remain in line with the economic outlook.[24]

Other data sources. We complement the AnaCredit credit registry with three additional data sources. First, to validate our PD-based measure of banks' expectations, we use bank-specific macroeconomic forecasts from Consensus Economics, supplemented with forecasts hand-collected from banks' research publications. Second, we obtain bank-level balance-sheet information from the European Central Bank's Individual Balance Sheet Items (iBSI) dataset, a confidential administrative dataset containing monthly information on the assets and liabilities of euro-area banks. Third, we use the European Banking Authority's EU-wide Transparency Exercise, which reports detailed information on the regulatory capital positions of 105 large European banking groups, including CET1 capital, regulatory capital ratios, and risk-weighted

[24]Source: European Banking Authority (2017), European Central Bank (2025). In practice, these models typically consist of two layers: a ranking layer, that assigns borrowers to risk grades, and a calibration layer, that maps each grade to a probability of default. Models are often based on logistic regressions estimated on a large vector of qualitative and quantitative predictors, selected via information criteria (Nehrebecka, 2017, Papanastasiou and Mok, 2017). The ECB has recently issued dedicated guidance on the use of machine-learning techniques (European Central Bank, 2025).

assets.

3.2 Identification Strategy: Estimating Bank-Level Belief Shocks

Having discussed our data, we now describe how we use it to estimate the response of bank lending to expectations.

The central challenge is that banks' expectations and lending respond jointly to borrower fundamentals. Deteriorating borrower prospects may lead a bank to revise its default forecasts upward, while also affecting demand for its loans and reducing its credit supply because of balance-sheet losses. A simple regression of lending on expected PDs would therefore confound the causal effect of beliefs with the common response to borrower conditions.

To address this challenge, we apply to the context of banks' expectations the within-borrower identification pioneered by Khwaja and Mian (2008) and Amiti and Weinstein (2018). Because most firms in our sample borrow from multiple banks, we can compare how different banks revise their default forecasts of the same borrower at the same date. This within-borrower comparison absorbs changes in lenders' common assessment of each borrower's fundamentals, allowing us to isolate the bank-specific component of forecast revisions.

Our strategy proceeds in two steps. First, we use fixed effects to decompose changes in expected PDs into bank-specific and borrower-specific components. Second, we use the bank-specific component in a local projection to trace out the dynamic response of bank lending to changes in banks' expectations.

Step 1: Decomposing belief changes. Consider a setting where firms $f = 1, \ldots, F$ borrow from multiple banks $b = 1, \ldots, B$.[25] Let $\widetilde{\mathbb{E}}_{bt} PD_{f,t+4}$ denote bank b's expectation in period t of the probability that firm f defaults over the next four quarters. This corresponds to the intermediary's expected portfolio default rate $\widetilde{\mathbb{E}}_{bt}\delta_{bt+1}$.[26] We define the change in these beliefs between $t-1$ and t as

$$\Delta\widetilde{\mathbb{E}}_{bt} PD_{f,t+4} \equiv \widetilde{\mathbb{E}}_{bt} PD_{f,t+4} - \widetilde{\mathbb{E}}_{b,t-1} PD_{f,t+3},$$

where the second term is bank b's one-year ahead expectation of default as of period $t-1$.

[25] Vice-versa, each bank b lends to multiple firms.

[26] In the model of Section 2, borrowers are ex-ante identical and only differ ex-post in their realized default outcomes. An intermediary's expected portfolio default rate therefore equals the default probability it assigns to each of its borrowers.

We can then decompose $\Delta\widetilde{\mathbb{E}}_{bt}PD_{f,t+4}$ into four components:

$$\Delta\widetilde{\mathbb{E}}_{bt}PD_{f,t+4} = \gamma_{ft} + \alpha_{bt} + \xi_{bf} + u_{bft}. \tag{21}$$

We refer to the first term, γ_{ft}, as the consensus change. By capturing forecast revisions shared across firm f's lenders, it absorbs changes in borrower fundamentals—such as leverage, profitability, or growth prospects—as well as macroeconomic or sectoral developments, insofar as banks incorporate these similarly into their forecasts.

The second term, α_{bt}, is our main object of interest. It captures the component of bank b's change in expectations at time t that is common across all of its borrowers. Because γ_{ft} absorbs each borrower's consensus change, we identify α_{bt} by comparing forecast revisions across banks lending to the same firms at the same date. As a result, α_{bt} measures how bank b's forecast revisions systematically differ from the consensus changes for the same borrowers. These bank-specific revisions may reflect differences across banks in how they process publicly available information or in their private information about aggregate conditions.

The third term, ξ_{bf}, captures persistent differences in average forecast revisions across bank–firm relationships. Because the left-hand side of (21) is a first difference, static level differences in reported PDs across relationships are already netted out and ξ_{bf} picks up any persistent bank-firm-specific drift in reported PDs.

Finally, the residual u_{bft} captures any remaining variation idiosyncratic to a bank-firm-time observations, such as, for example, private information bank b obtains about firm f or noise in the bank's internal ratings model.

In practice, we recover each of these components by estimating (21) as a fixed-effects regression, with γ_{ft}, α_{bt} and ξ_{bf} modeled, respectively, as a firm×time, bank×time, and bank×firm fixed effects. As in Amiti and Weinstein (2018), the connected structure of the bank-firm network delivers the full set of fixed effects up to a normalization.[27]

Variation in banks' expectations. Even after accounting for firm–quarter fixed effects, we observe substantial variation in banks' expectations. As Table B.2 in Appendix B shows, more than half of the variance in quarterly PD updates reflects differences across banks assessing the

[27] De Jonghe and Lewis (2026) show that bank and firm fixed effects estimated from loan volumes need not isolate credit supply and demand shocks in the presence of heterogeneity across lending relationships, as firms may shift demand toward particular banks. This concern is less relevant in our setting, as we observe banks' forecasts of the same borrower-level default probability and decompose their revisions, rather than infer supply and demand shocks from equilibrium lending.

same borrower at the same date.[28] This within-borrower variation is roughly evenly split between the extensive margin of whether banks update their PD forecasts and the intensive margin of how much they revise them. Table B.3 shows that banks update reported PDs once every 2.4 quarters on average, with slightly more than half of the variance in whether a PD changes occurring across lenders to the same borrower in the same quarter. Our empirical approach uses cross-sectional variation stemming from the bank-specific component of these within-firm forecast revisions, α_{bt}. As Figure B.2 shows, we estimate this component using 5,810 (2,150) shared borrowers in the average (median) bank–quarter, comparing that bank's forecast revisions with those of 80 (93) other lenders. The resulting estimates, $\widehat{\alpha}_{bt}$, have a standard deviation of 0.12 percentage points. To put this magnitude in perspective, Figure B.1 shows the rise in realized corporate default rates in Germany and France during the 2008 Global Financial Crisis (GFC). A one-standard-deviation change in $\widehat{\alpha}_{bt}$ amounts to roughly one-tenth of the increase in Germany's aggregate default rate during the GFC and one-sixth of the corresponding increase in France.

At this stage, Equation (21) is purely a statistical decomposition of the panel and does not, by itself, attach a structural interpretation to its estimates $\widehat{\gamma}_{ft}, \widehat{\alpha}_{bt}$, and $\widehat{\xi}_{bf}$. In the next step, we lay out the identifying assumptions under which $\widehat{\alpha}_{bt}$ can be interpreted as an idiosyncratic bank-level shock to expectations.

Step 2: The response of lending to beliefs. Our goal is to give $\widehat{\alpha}_{bt}$ a causal interpretation and trace the dynamic response of bank lending to beliefs. To this end, consider the following local projection (Jordà, 2005):

$$\Delta \log y_{bt+h} = \delta_{th} + \omega_{bh} + \beta_h \widehat{\alpha}_{bt} + \Xi_h' \boldsymbol{X}_{bt-1} + e_{bth}, \tag{22}$$

where $\Delta \log y_{bt+h} \equiv \log y_{bt+h} - \log y_{bt-1}$ is the h-period-ahead cumulative log change in bank b's lending, δ_{th} and ω_{bh} are, respectively, time and bank fixed effects, and $\boldsymbol{X}_{bt-1}$ is a vector of lagged bank-level controls.

To give the sequence of $\{\beta_h\}_{h=0}^{H}$ coefficients a causal interpretation, our identifying assumption is that, after conditioning on the controls included in Equation (22), $\widehat{\alpha}_{bt}$ is unrelated to other unobserved determinants of lending.

By focusing on within-borrower variation, our first step purges $\widehat{\alpha}_{bt}$ of the component of forecast revisions common across a borrower's lenders. First, this separates bank-specific shifts in expectations from forecast revisions driven by changes in borrower outlook that are reflected in

[28] Intuitively, the corresponding share is smaller for PD *levels*, where differences across lenders within firm–quarters account for roughly 17.5% of total variance.

the consensus and may affect lending through other channels, such as loan demand or banks' balance sheets. Second, it accounts for differences in borrower composition across banks, such as differences in their exposure to sectoral or regional economic fluctuations. Moreover, the time fixed effect δ_{th} in (22) absorbs aggregate shocks—such as recessions or changes in the stance of monetary policy—that may simultaneously move banks' expectations and their lending. Finally, the inclusion of lagged bank-level controls $\boldsymbol{X}_{bt-1}$ in (22) allows us to flexibly absorb variation in $\widehat{\alpha}_{bt}$ that is correlated with observable bank fundamentals, such as recent trends in risk-taking, deposits, or profitability. We will discuss remaining potential threats to identification momentarily.

Note that we remain agnostic about where movements in $\widehat{\alpha}_{bt}$ come from. They may reflect differences in the information banks hold about the economy, or heterogeneity in how banks interpret the same information. All we require is that $\widehat{\alpha}_{bt}$ isolates variation in banks' expectations of future returns that is orthogonal to balance sheets. These different sources are hence observationally equivalent for our purposes.

Our bank-specific shock $\widehat{\alpha}_{bt}$, together with time fixed effects, allows us to estimate the lending response from cross-sectional variation in expected borrower default probabilities. Through the lens of the model in Section 2, we interpret this variation as differences in $\mathcal{S}$, and hence in intermediaries' expected portfolio default rates. Under our identifying assumption, the coefficients $\{\beta_h\}_{h=0}^{H}$ therefore help discipline the partial equilibrium lending sensitivity $\partial k'(n, \mathcal{S})/\partial \mathcal{S}$, plotted in panel (c) of Figure 1.

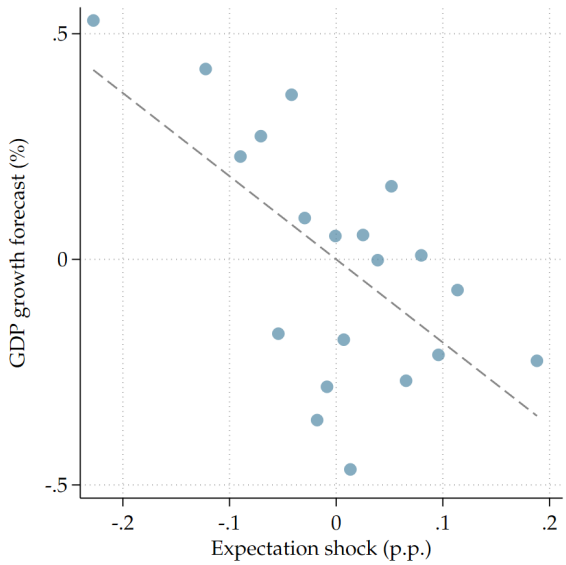
Potential threats to identification. Before presenting our main empirical results, we discuss some potential threats to identification.

Strategic reporting. A first concern is that, since $\widehat{\alpha}_{bt}$ is constructed from regulatory microdata, it may capture reporting incentives rather than banks' genuine expectations. To address this concern, we show that our estimated $\widehat{\alpha}_{bt}$ correlates with another measure of banks' expectations that comes from a different data source and carries no regulatory implications: banks' research-department forecasts of Eurozone GDP growth. Panel (a) of Figure 5 plots $\widehat{\alpha}_{bt}$ against these GDP forecasts residualized by time fixed effects. The two measures are negatively correlated: banks that are more pessimistic according to our measure also forecast lower GDP growth, suggesting that $\widehat{\alpha}_{bt}$ captures genuine differences in banks' economic outlook.[29]

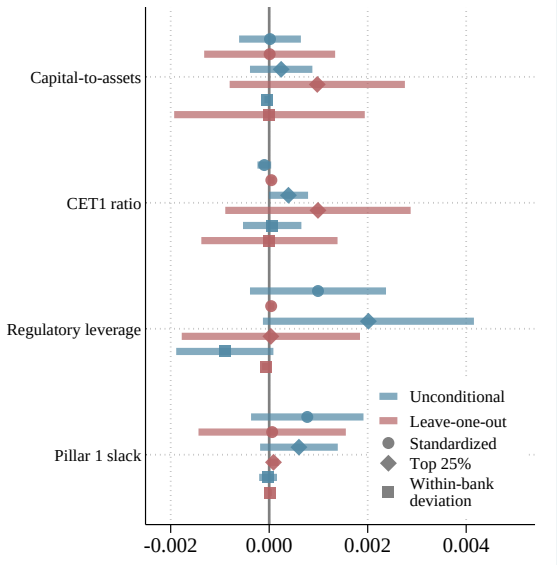
However, since reported PDs enter capital requirements, banks may still face different reporting incentives depending on their capital position. We therefore examine whether banks' regulatory capital positions are associated with their reported PD revisions. To do so, we

[29] The correlation coefficient is -0.19 and is statistically significant at the 1% confidence level.

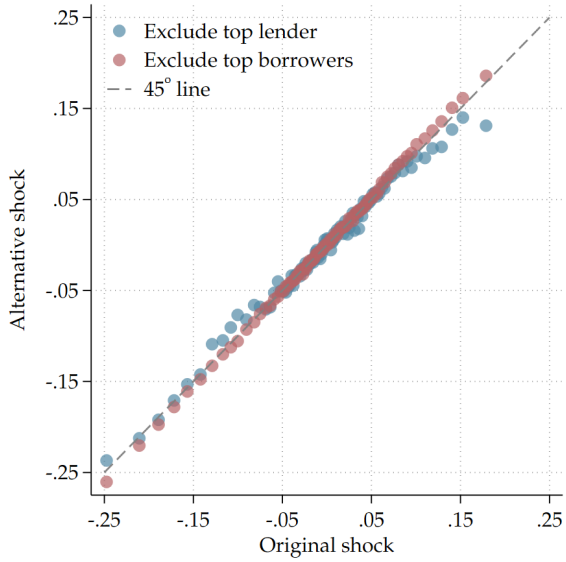
Figure 5 – Expected default probabilities, beliefs, and balance sheets



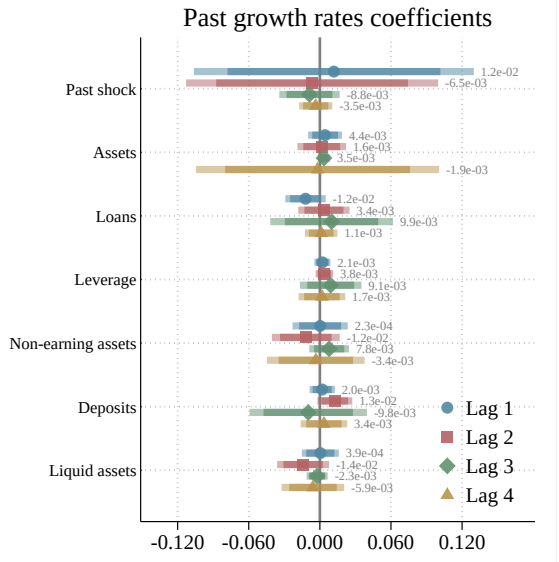
(a) Belief shocks and GDP forecasts



(b) Bank leverage and expected PDs



(c) Belief shocks and reverse causality



(d) Belief shocks and observables

Notes: panel (a) is a binned scatter plot of our estimated belief shock $\widehat{\alpha}_{bt}$ (x-axis) against banks research-department forecasts of Eurozone GDP growth (y-axis). Panel (b) plots the coefficients β_1 and β_2 from estimating (23) and (23) in the main text. Panel (c) is a binned scatter plot of our original $\widehat{\alpha}_{bt}$ measure against two alternative measures of $\widehat{\alpha}_{bt}$ obtained by re-estimating (21) dropping either the bank a given firm borrows the most from (blue dots) or the firms in the top 5% of a bank's exposures (red dots). Panel (d) plots the coefficients of a regression of $\widehat{\alpha}_{bt}$ on four lags of $\widehat{\alpha}_{bt}$ itself, as well as growth rates of assets, loans, leverage, non-earning assets, deposits, and liquid assets. In both panels (b) and (d) light-shaded and dark-shaded lines represent, respectively, 95% and 90% confidence bands.

regress quarterly PD revisions for a given bank-firm pair, $\Delta\widetilde{\mathbb{E}}_{bt}PD_{ft+4}$, on a measure of the bank's capital position, including firm–time fixed effects:

$$\Delta\widetilde{\mathbb{E}}_{bt}PD_{ft+4} = \gamma_{ft} + \beta_1 \text{Capital}_{bt} + u_{bft}. \tag{23}$$

We consider four different measures of banks' regulatory capital positions: the capital-to-assets ratio, the CET1 ratio, the regulatory leverage ratio, and the Pillar 1 capital slack.[30] We include each measure separately in (23) and estimate three specifications, using: (i) the continuous measure, standardized to have mean zero and standard deviation one; (ii) the deviation of bank b's period-t value from that bank's average over all periods in the sample, in the spirit of Ottonello and Winberry (2020); and (iii) an indicator equal to one when bank b is in the top quartile of the measure's distribution in that quarter and zero otherwise.

To further examine whether banks' capital positions are associated with how they revise reported PDs *conditional* on news about a borrower, we also consider a different specification of (23), where we include an interaction of Capital_{bt} with a leave-one-out measure of changes in a borrower's reported default probability:

$$\Delta\widetilde{\mathbb{E}}_{bt}PD_{ft+4} = \gamma_{ft} + \beta_2 \text{Capital}_{bt} \times \overline{\Delta PD^e}_{-bft+4} + \chi \text{Capital}_{bt} + \zeta\overline{\Delta PD^e}_{-bft+4} + u_{bft}, \tag{24}$$

where $\overline{\Delta PD^e}_{-bft+4} = \frac{1}{N_{ft}-1}\sum_{b'\neq b}\Delta\widetilde{\mathbb{E}}_{b't}PD_{ft+4}$ is the average PD revision reported by firm f's lenders other than bank b and N_{ft} denotes the number of lenders for that firm in quarter t.

Panel (b) of Figure 5 reports the estimated $\widehat{\beta}_1$ and $\widehat{\beta}_2$ coefficients for the four capital measures and three specifications described above. The point estimates are both statistically insignificant and economically small, providing little evidence that capital-related reporting incentives systematically affect banks' reported PD revisions.[31]

Reverse causality. A second potential threat to our empirical strategy is that banks may revise their PDs in anticipation to changes in their own credit supply, so that $\widehat{\alpha}_{bt}$ mechanically

[30] The capital-to-assets ratio is computed as the ratio of capital and reserves to total assets. The CET1 ratio is defined as Common Equity Tier 1 capital divided by risk-weighted assets. The regulatory leverage ratio is defined as Tier 1 capital divided by total regulatory exposure. Finally, the Pillar 1 capital slack is constructed as the smallest percentage point surplus above the three Pillar 1 risk-based capital requirements, i.e., the minimum among CET1 ratio$_{bt}$ − 4.5%, Tier 1 ratio$_{bt}$ − 6%, and Total capital ratio$_{bt}$ − 8%.

[31] Existing evidence suggests that misreporting of PDs takes the form of static *level* effects concentrated upon origination rather than a revision of PDs on existing loans over time (Behn et al., 2022). This form of misreporting is not a threat to our empirical analysis, as our identification exploits within-bank variation in PDs over time. As such, loan originations are excluded by design and static misreporting is differenced out by fixed effects. Supervisory oversight also limits the scope for dynamic misreporting. Regulation requires PDs to perform well out-of-sample, with persistent under- or over-prediction triggering supervisory intervention. In line with this, Behn et al. (2022) document a sharp decline in misreporting over time: the bias in internal PD estimates fell from 1.6 to 1.0 percentage points between 2008 and 2012.

captures future changes in bank lending. To address this, we re-estimate (21) dropping the bank-firm relationships most exposed to this concern. First, for each firm we exclude the bank that supplies it the most credit. Second, for each bank we exclude its top 5% of borrowers by outstanding exposure. In panel (c) of Figure 5 we then show a binned scatter plot of our original $\widehat{\alpha}_{bt}$ against the two measures just described. The points cluster around the 45-degree line, showing that the resulting estimates are virtually unchanged.[32] This helps mitigate concerns that our bank-time component reflects banks revising PDs in response to their own credit-supply decisions.

Correlation with balance sheets. Another concern is that $\widehat{\alpha}_{bt}$ correlates with changes in banks' balance sheets, so that the estimates from (22) could partly capture the response of lending to balance-sheet dynamics. To address this concern, we test whether our estimated $\widehat{\alpha}_{bt}$ is predictable from past developments in banks' balance sheets. In particular, we regress $\widehat{\alpha}_{bt}$ on four lags of the growth rates of a bank's total assets, total loans, leverage, non-earning assets, deposits, and liquid assets. We also include four lags of $\widehat{\alpha}_{bt}$ itself to examine whether our bank-level expectation shock captures innovations in beliefs or partly reflects predictable revisions.[33]

Panel (d) of Figure 5 plots the estimated coefficients. We find no evidence that banks become systematically more optimistic or pessimistic relative to the consensus following expansions or contractions in their balance sheets: the coefficients on lagged balance sheet variables are economically small and statistically insignificant. The coefficients on the shock's own lags are likewise small and statistically insignificant, consistent with our measure capturing innovations in beliefs.

Specialization and information. Finally, another potential challenge to our identification is that banks' expectations may respond differently to news about the sectors, regions, or types of borrowers in which they specialize. Our estimated $\widehat{\alpha}_{bt}$ may then correlate with contemporaneous balance-sheet changes, as news that sways a bank's expectations more strongly may also have a greater effect on the performance of its loan portfolio.

We assess this concern by examining whether bank-specific forecast revisions are shared across different groups of borrowers. First, we estimate $\widehat{\alpha}_{bt}$ separately using different borrower samples. Panels (a) and (b) of Figure B.3 in Appendix B compare the estimates obtained using manufacturing firms with those from firms in the trade sector and the transport sector, respectively. Panel (c) compares estimates from borrowers above and below the median baseline PD, while panel (d) compares those from borrowers with secured and unsecured loans. Across

[32] The R-squared of the underlying regressions is 0.98 for the case in which we exclude the banks' largest borrowers, and 0.88 for the case in which we exclude each borrower's largest lender.

[33] Since the first-step decomposition identifies α_{bt} up to bank- and quarter- specific normalizations, the regression includes time and bank fixed effects.

all four comparisons, the estimates lie close to the 45-degree line, suggesting that the shifts in expectations captured by $\widehat{\alpha}_{bt}$ are not confined to a particular lending segment.

We extend these comparisons using more granular sectoral and regional classifications to examine whether the bank-specific component of forecast revisions is homogeneous across borrowers. To do so, we re-estimate (21), replacing bank–time fixed effects with bank–time–sector fixed effects and, in a separate specification, bank–time–region fixed effects.[34] Figure B.4 in Appendix B plots the results. The estimates align closely across sectors and regions, supporting the homogeneity restriction on α_{bt} in our baseline decomposition (21).

Finally, we test whether banks revise PDs differently for borrowers mo re dependent on them for credit following the α_{bt} shock. To do so, we run (22) using the cumulative change in the reported PD as the dependent variable and adding an interaction between α_{bt} and an indicator for whether the borrower is among the bank's largest 10% of exposures. Panel (b) of Figure B.5 in Appendix B shows no evidence of a differential response for these borrowers.

3.3 The Response of Lending to Expected Returns

Having described our identification approach, we now turn to estimating the response of bank lending to expected returns.

In our baseline specification of the local projection (22) we partial out the effect of PD revisions on banks' regulatory capital requirements, which are defined as a proportion of a bank's risk-weighted assets. Since our microdata include the same PDs used to calculate the risk weights on banks' loans, we control for the change in risk-weighted assets implied by observed PD revisions, holding loan amounts fixed at their pre-shock values.[35] We rescale $\widehat{\alpha}_{bt}$ so that a unit innovation corresponds to a one-percentage-point increase in bank b's expected portfolio return. The expected gross return on one euro of lending is $(1 - PD) \times R^{\ell} + PD \times \text{RecoveryRate}$, where R^{ℓ} is the average gross loan rate and RecoveryRate is the average fraction of principal recovered upon default. We therefore convert PD revisions into changes in expected returns using the difference between the average loan and recovery rates.[36]

Panel (a) of Figure 6 plots the estimated coefficients $\{\widehat{\beta}_h\}_h$, with standard errors clustered at the bank level. The estimates are flat and insignificant for $h < 0$, showing no evidence of differential pre-shock behavior in bank lending. Following a shock that increases expected

[34] We define sectors at the two-digit level and regions at the NUTS 2 level.

[35] See Appendix B for more details on how we construct the change in risk-weighted assets.

[36] In particular, given an average interest rate in our sample of 0.0317 and an average loss given default on corporate loans reported in Galow et al. (2024) of 0.215, a one-percentage-point decrease in PD translates into a 0.247 increase in the expected portfolio-wide return.

future returns by one percentage point (thus decreasing expected default probabilities) we find that banks raise lending sharply, with the total stock of loans increasing by as much as 12% five quarters after the shock.[37]

Our theory predicts a heterogeneous pass-through from expectations to lending, with a more muted response for intermediaries closer to their leverage constraint. We examine this prediction using bank leverage as a proxy for the tightness of financial constraints. Specifically, we augment the baseline local projection (22) with an interaction between our expectation shock and pre-shock bank leverage:

$$\Delta \log y_{bt+h} = \delta_{th} + \omega_{bh} + \beta_h \widehat{\alpha}_{bt} + \phi_h \widehat{\alpha}_{bt} \times \text{Lev}_{bt-1} + \zeta \text{Lev}_{bt-1} + \Xi_h' \boldsymbol{X}_{bt-1} + e_{bth}, \tag{25}$$

where Lev_{bt-1} is computed as the ratio of assets to equity of bank b in the quarter preceding the shock, and normalized to have unitary standard deviation.

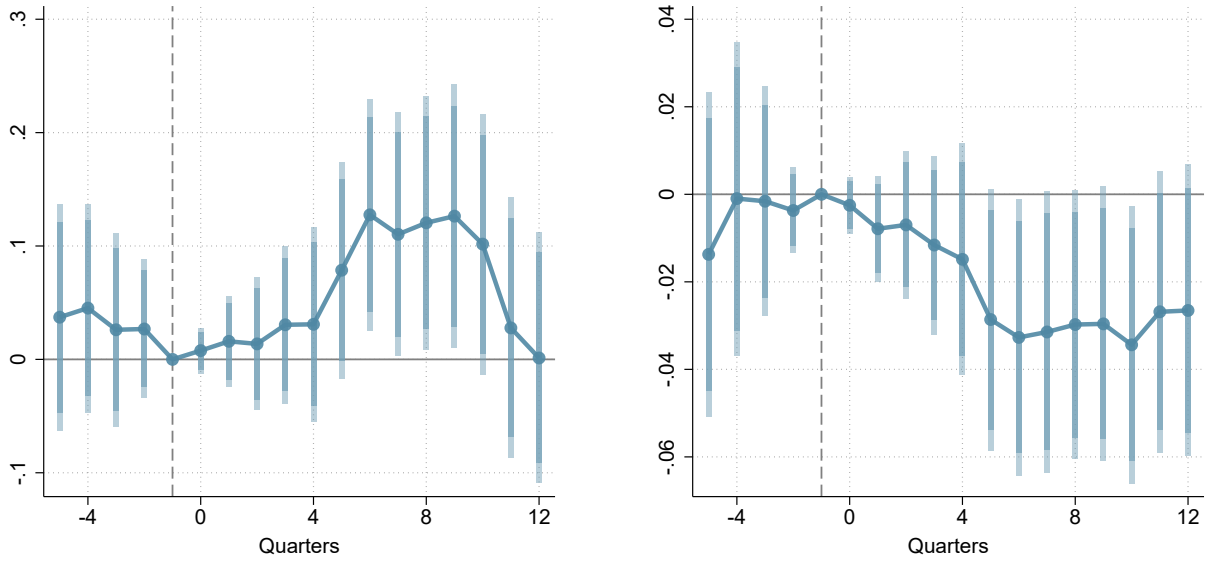
Panel (b) of Figure 6 plots the estimated interaction coefficients $\{\widehat{\phi}_h\}_h$. As before, we find no evidence that lending behavior before the shock differs systematically with bank leverage, with the estimates for $h < 0$ small in magnitude and statistically insignificant. Following the shock, the coefficients turn significantly negative, indicating that more leveraged banks increase lending by less after an increase in expected portfolio returns. The magnitudes are economically significant: a one-standard-deviation increase in leverage reduces the lending response to $\widehat{\alpha}_{bt}$ by 3 percentage points, roughly one quarter of the average effect we estimate in panel (a).

Robustness and additional results. In Appendix B we report some additional results, together with a series of robustness checks for the estimates in Figure 6. In panel (a) of Figure B.6, we show that our results are unchanged when we re-estimate (22) on the subsample of banks with above-median regulatory capital headroom, for which strategic reporting incentives should be weaker and PD-induced changes in capital requirements less relevant. Panel (b) shows that our estimates are qualitatively unchanged when we include country×quarter fixed effects. In panels (a) and (b) of Figure B.7 we show that the estimates change little when we exclude 2020Q1–2021Q1 or restrict the sample to 2019Q3–2022Q4, respectively, suggesting that the results are stable across sample periods. Finally, panels (a) and (b) of Figure B.8 respectively show that our estimates are unchanged when we use a lag-augmented local projection specification (Montiel Olea and Plagborg-Møller, 2021), or when we control for four lags of pre-shock changes in leverage.

We also examine how much substitution across lenders dampens the effect of bank-level

[37] Note that while the shock $\widehat{\alpha}_{bt}$ is serially uncorrelated, it induces a persistent response in the *level* of reported PDs, as shown in panel (a) of Figure B.5 in Appendix B.

Figure 6 – The response of bank lending to expected returns



(a) Average response **(b)** Leverage heterogeneity

Notes: panel (a) plots the estimated β_h coefficients from the local projection in (22). The y-axis denotes the percentage change, relative to the quarter before the belief shock. Panel (b) plots the estimated ϕ_h coefficients from the local projection in (25). The shock $\widehat{\alpha}_{bt}$ is normalized to induce a one-percentage-point increase in expected portfolio returns. In both panels, standard errors are clustered at the bank level. Light and dark bars represent, respectively, 95% and 90% confidence bands.

expectation shocks on firms' total borrowing. In panel (a) of Table B.4 we estimate the lending response at the bank-firm level, including firm–time fixed effects to control for borrower demand as in Khwaja and Mian (2008). Panel (b) reports the corresponding estimates from a regression run at the firm-level. The firm-level coefficients remain statistically significant and are only 10% smaller in magnitude than their bank–firm counterparts, suggesting that substitution across lenders offsets only a small part of the effect of bank-level expectation shocks on firms' borrowing. In Table B.5 we estimate the response of firm-level employment to the average bank-level shock a firm is exposed to. Firms whose lenders become more optimistic significantly increase employment for up to two years following the shock. Finally, we examine the response across loan maturities and borrower types. Panel (a) of Figure B.9 shows that long-term loans respond more strongly than short-term loans, while panel (b) shows that both corporate and household loans respond to the shock, with comparable magnitudes. Panel (c) plots the response of the loans-to-equity ratio to the expectation shock.

Taking stock. The results in this section help inform one of the key moments from our theory: the partial equilibrium pass-through from financial intermediaries' expectations of future re-

turns to lending. Our findings suggest that expectations are central to understanding the role of the financial sector in aggregate fluctuations. Next, we turn to studying how these expectations respond to aggregate shocks.

4 How Banks' Expectations Respond to Aggregate Shocks

Having estimated how bank lending responds to expectations of future returns, we now turn to estimating the second of the two moments highlighted by our theory: how these expectations themselves respond to aggregate shocks.

To do so, we study the dynamic response of banks' forecast errors to macroeconomic shocks, following an approach popularized by Coibion and Gorodnichenko (2012), Angeletos et al. (2021).

Defining forecast errors. We first construct our measure of forecast error. In particular, we denote by $\mathcal{E}_{bft}$ the error that bank b makes in forecasting the default probability of borrower f at date t and define it as:

$$\mathcal{E}_{bft} = \widetilde{\mathbb{E}}_{bt} PD_{ft+4} - \mathbb{D}_{ft+4}, \tag{26}$$

where $\widetilde{\mathbb{E}}_{bt} PD_{ft+4}$ is the default probability forecast by bank b at date t, and $\mathbb{D}_{ft+4}$ is an indicator equal to one if firm f defaults between t and $t+4$ and zero otherwise. Under this convention, $\mathcal{E}_{bft} > 0$ corresponds to the bank having been ex-post overly pessimistic about firm f, and $\mathcal{E}_{bft} < 0$ to it having been overly optimistic.

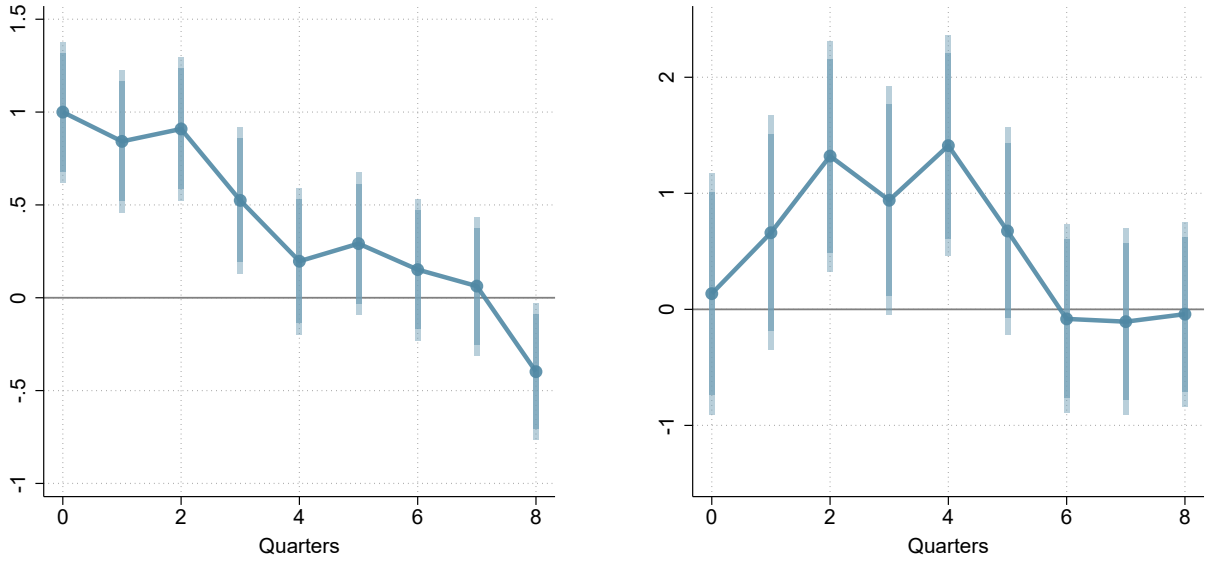
We then aggregate these errors to the bank level by averaging across the borrowers in each bank's loan portfolio:

$$\mathcal{E}_{bt} = \frac{1}{N_{bt}} \sum_{f'=1}^{N_{bt}} \mathcal{E}_{bf't}, \tag{27}$$

where N_{bt} denotes the number of borrowers in bank b's loan book at date t. A positive value of $\mathcal{E}_{bt}$ therefore means that the bank overestimated the default rate subsequently realized among its borrowers.

Empirical strategy. We next estimate the dynamic response of forecast errors $\mathcal{E}_{bt}$ to aggregate shocks. To do so, for each horizon $h \geq 0$, we estimate the following local projection (Jordà,

Figure 7 – The response of banks' expectations to aggregate shocks



(a) Realized default rates **(b)** Forecast error

Notes: panel (a) plots the estimated β_h coefficients from the local projection in (28) using the bank-level average default rate $\mathbb{D}_{ft+4}$ as the outcome variable. The y-axis denotes the percentage-point change. Panel (b) plots the estimated β_h coefficients from the local projection in (28) using the default forecast error $\mathcal{E}_{bt}$ as the outcome variable. The y-axis denotes the percentage-point change. We use the Angeletos et al. (2020) main business cycle shock as the measure of the ε_t^s shock, and normalize it to induce a 1 percentage point increase in realized default rates on impact. In both panels, standard errors are clustered at the bank level. Light and dark bars represent, respectively, 95% and 90% confidence bands.

2005):

$$\mathcal{E}_{bt+h} = \delta_{bh} + \beta_h \times \varepsilon_t^s + \Xi_h \mathbf{Z}_t' + u_{bt+h}, \tag{28}$$

where ε_t^s denotes the aggregate shock and δ_{bh} is a bank fixed effect. $\mathbf{Z}_t'$ is a vector of aggregate controls that includes four lags of the shock ε_t^s itself, as well as four lags of the ECB deposit rate, (the log of) euro area GDP, (the log of) inflation, and (the log of) the Eurostoxx price index. We cluster standard errors at the bank level.

Following Angeletos et al. (2021) we use the main business cycle shock of Angeletos et al. (2020) as our baseline measure of aggregate shock.[38] In Appendix C we show our results are robust to using the oil supply news shock proposed by Känzig (2021).

Expectations overreact to shocks. Figure 7 plots the responses of realized defaults and banks' forecast errors to a contractionary aggregate shock. We normalize the shock to raise the realized

[38]We apply the procedure in Angeletos et al. (2020) to construct a quarterly series of the main business cycle shock for the euro area.

one-year default rate by one percentage point on impact.

Panel (a) of Figure 7 shows the response of realized defaults over the following year, by estimating (28) using the bank-level average default rate $\mathbb{D}_{ft+4}$ as the outcome variable. The shock induces a persistent increase in realized defaults, with the effect waning after about five quarters.

Panel (b) of Figure 7 shows the response of forecast errors, by plotting the estimated coefficients $\widehat{\beta}_h$ from (28). Banks' expectations overreact to aggregate shocks. Following a contractionary shock, forecast errors turn systematically positive, implying banks' expectations of default probabilities increase by more than subsequent realized default rates do. The magnitude of this overreaction is within the range of estimates that Bordalo et al. (2019, 2026) find for financial analysts and managers. Following a shock raising realized default probabilities by one percentage point, we find that forecast errors increase by up to 1 percentage point after two quarters, with the response remaining significantly positive for up to five quarters.

The overreaction we find is consistent with the way banks' default probability models are constructed. These models are estimated on a large cross-section of borrowers, but often rely on a relatively short time series.[39] Recent observations therefore account for a large share of the data used to estimate how default rates respond to economic conditions. This may lead banks to place excessive weight on recent observations, a mechanism at the core of leading theories of belief overreaction (Bordalo et al., 2018).

Appendix C presents additional evidence consistent with overreaction of expectations, showing that default forecast errors are negatively correlated with forecast revisions.

Taking stock. The results in this section show that banks' expectations overreact to aggregate shocks: following a contractionary shock, banks revise their default expectations by more than realized default rates and remain overly pessimistic for several quarters. Through the lens of the model in Section 2, this places financial intermediaries in the overreaction region, where $\chi > 1$. Together with the response of bank lending to expected returns estimated in Section 3, these results help discipline the two moments highlighted by our theory. In the next section, we calibrate the model of Section 2 to these estimates and show that matching the microdata leads to different predictions for the effectiveness of policy during crises and the buildup of financial fragility before them.

[39] The ECB guide requires models to cover at least five years of data.

5 Quantitative Model

We now turn to studying what the two moments we estimated in the microdata imply for aggregate dynamics and policy. To do so, we calibrate the heterogeneous-intermediary model from Section 2 to our empirical evidence on bank lending and expectations. In general equilibrium, overreaction of expectations and heterogeneity interact, allowing the calibrated model to generate large recessions following financial crises, consistent with the aggregate data, while matching our micro evidence on intermediaries' forward-looking behavior. Matching the microdata yields two main results. First, it has important implications for the effectiveness of policy *during* crises, making recapitalizations less powerful at stimulating investment and output. Second, it has also implications for the buildup in fragility *before* crises, allowing the model to rationalize the empirical evidence that larger pre-crisis expansions are followed by deeper recessions.

5.1 Calibration

We calibrate the model from Section 2 at a quarterly frequency. Table 1 summarizes the parameter values and targets. We describe our calibration strategy in detail below.

Household preferences. Households have isoelastic instantaneous utility that is separable in consumption and hours worked, $v(C_t, H_t) = \frac{C_t^{1-\gamma^{\text{HH}}}}{1-\gamma^{\text{HH}}} - \zeta \frac{H_t^{1+\varphi}}{1+\varphi}$. We set the household's discount factor β^{HH} so that the annualized real deposit rate is 4% in steady state. We set the inverse elasticity of intertemporal substitution γ^{HH} to 1, a standard value in the literature. We set the inverse Frisch elasticity to $\varphi = 1$ as in Kaplan et al. (2018). We calibrate the labor-disutility parameter jointly with the steady state so that hours worked equal one-third of the normalized time endowment.

Production technology. As described in Section 2, final goods are produced with a Cobb-Douglas technology. We normalize effective steady state total factor productivity to 1 and set the parameter $\alpha = 0.66$ implying a labor share of 66%.

Capital-good producers face standard quadratic adjustment costs à la Lucas and Prescott (1971):

$$\Psi(I_t, K_t) = \frac{\psi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t.$$

Table 1 – Model calibration

Parameter	Description	Value	Calibration
	Externally calibrated parameters		
β^{HH}	Household discount factor	0.990243	4% annual interest rate
γ^{HH}	Inverse EIS	1	Standard
φ	Inverse Frisch elasticity	1	Kaplan et al. (2018)
ζ	Labor disutility	6.897	Hours worked $= 1/3$
μ_w	Wage markup	1.1	Auclert et al. (2021)
κ_w	Wage-adjustment coefficient	0.1	Auclert et al. (2021)
α	Labor share in production	0.66	Labor share $= 2/3$
δ	Depreciation rate	0.0149	Investment-to-output $= 20$
ψ	Investment adjustment cost	7	Chodorow-Reich (2025)
ϕ_π	Taylor coefficient	1.5	Standard
$\underline{d}$	Dividend lower bound	0	No equity issuance
β	Intermediary discount factor	$\beta^{\text{HH}}/\varrho$	Normalization
	Internally calibrated parameters		
θ	Divertible share of assets	0.134563	See main text
n^{entry}	Entrant net worth	1.345952	See main text
ϱ	Intermediary survival probability	0.9908	See main text
γ	Intermediary's risk aversion	0.1193	See main text
σ_ε	Idiosyncratic return std. dev.	0.026	See main text
χ	Diagnosticity parameter	2.484	Match Figure 7
η	Memory decay parameter	0.922	Match Figure 7

Notes: the calibration is quarterly. The expectation formation process (29) is expressed such that $\chi = 1$ recovers the FIRE benchmark.

We set ψ to a quarterly value of 7, within the range of the estimates in Chodorow-Reich (2025), and the quarterly depreciation rate to $\delta = 0.0149$ to match an investment-to-output ratio of 20%.

Nominal rigidities. To capture the aggregate demand effects of investment, we introduce nominal rigidities in the form of sticky wages. The representative household supplies labor to a continuum of monopolistically competitive labor union that face quadratic utility costs to wage adjustments. This gives rise to a New Keynesian Phillips curve of the form:[40]

$$\log(1+\pi_t^w) = \beta^{\text{HH}}\mathbb{E}_t\left[\log(1+\pi_{t+1}^w)\right] + \kappa_w\left[\zeta H_t^{1+\varphi} - \frac{w_t H_t C_t^{-\gamma^{\text{HH}}}}{\mu_w}\right],$$

[40] Appendix D provides more detail on the derivation of the Phillips curve.

where π_t^w denotes net nominal wage inflation, w_t is the real wage, and μ_w is the desired wage markup.

We set the desired wage markup to $\mu_w = 1.1$ and the slope of the wage Phillips curve $\kappa_w = 0.1$, in line with Auclert et al. (2021).

We also assume that the monetary authority sets nominal interest rates according to a standard Taylor rule:

$$1 + i_t = (1 + \bar{r})(1 + \pi_t)^{\phi_\pi}.$$

Where $\bar{r}$ is the steady state net real interest rate, π_t is net inflation, and we consider a steady state with zero inflation. We set the Taylor coefficient ϕ_π to 1.5, a standard value in the literature.

Financial intermediaries. To calibrate the standard deviation of idiosyncratic returns σ_ε we use banks' financial statements. We construct a quarterly measure of bank-level returns on their portfolio of loans, regress it on bank and time fixed effects and set σ_ε to the standard deviation of the residuals obtained from this procedure. Our baseline calibration sets $\sigma_\varepsilon = 0.026$. See Appendix D.1 for more details. We further impose a no-equity-issuance constraint and set the lower bound on dividends $\underline{d}$ to 0.[41] We also set steady-state government transfers to active intermediaries $\bar{T}^b$ to zero.

We jointly calibrate intermediaries' risk aversion γ, survival probability ϱ, divertible share of assets θ, and entrants' endowment n^{entry} to four targets.

The first target is the 12% increase in lending per percentage-point increase in expected excess returns estimated in Figure 6. We construct the model counterpart using the belief shifted $\mathcal{S}$ to trace out the lending policy function as expected excess returns change, holding current net worth, aggregate prices and beliefs, and the leverage constraint fixed. This yields the partial-equilibrium lending sensitivity $\partial k'(n, \mathcal{S})/\partial \mathcal{S}|_{\mathcal{S}=0}$ introduced in Section 2. To express this sensitivity in the same units as the empirical estimate, we rescale $\mathcal{S}$ so that a unit increase raises expected excess returns by one percentage point and normalize each intermediary's lending response by its steady-state capital holdings. We then match the average response across intermediaries to this target.

The remaining targets are an average leverage ratio of 8.2, corresponding to the average loans-to-equity ratio in our sample; an average elasticity of lending to net worth of one, in line with the estimates in Bellifemine et al. (2026); and a capital-to-annual-output ratio of 3.38,

[41] In steady state intermediaries' dividends account for approximately 1.5% of GDP.

corresponding to the euro-area average over 2018-2025. We normalize intermediaries' discount factor so that $\beta\varrho = \beta^{\text{HH}}$.

Expectation formation. Finally, to match our empirical estimates of the response of banks' forecast errors to aggregate shocks from Figure 7, we specify the distant-memory diagnostic expectations framework proposed in Maxted (2024), Bianchi et al. (2024). In particular, as in Section 2 we assume that financial intermediaries form expectations about exogenous processes and given these expectations form model-consistent forecasts of endogenous objects.[42] We focus on the case in which exogenous variables follow AR(1) processes in logs $x_t = \rho x_{t-1} + u_t$. We specify geometrically declining memory weights for the reference level of x_t used to characterized representativeness, so that intermediaries' expectations about x_t are characterized by:

$$\widetilde{\mathbb{E}}_t x_{t+1} = \mathbb{E}_t\, x_{t+1} + (\chi - 1)\left(\mathbb{E}_t\, x_{t+1} - (1-\eta)\sum_{j=1}^{\infty} \eta^{j-1}\, \mathbb{E}_{t-j}\, x_{t+1}\right), \tag{29}$$

where $\widetilde{\mathbb{E}}_t(\cdot)$ denotes the diagnostic expectations operator, $\mathbb{E}_t(\cdot)$ is the rational expectations operator, χ governs the degree of diagnosticity (with $\chi = 1$ recovering the FIRE benchmark), and η governs its persistence.

We then set $\chi = 2.484$ and $\eta = 0.922$ to minimize the distance between the model-implied forecast error response to an aggregate productivity shock and its empirical counterpart we estimate in Figure 7.[43] See Appendix D.2 for more details.

5.2 Matching the Micro and Macro Evidence

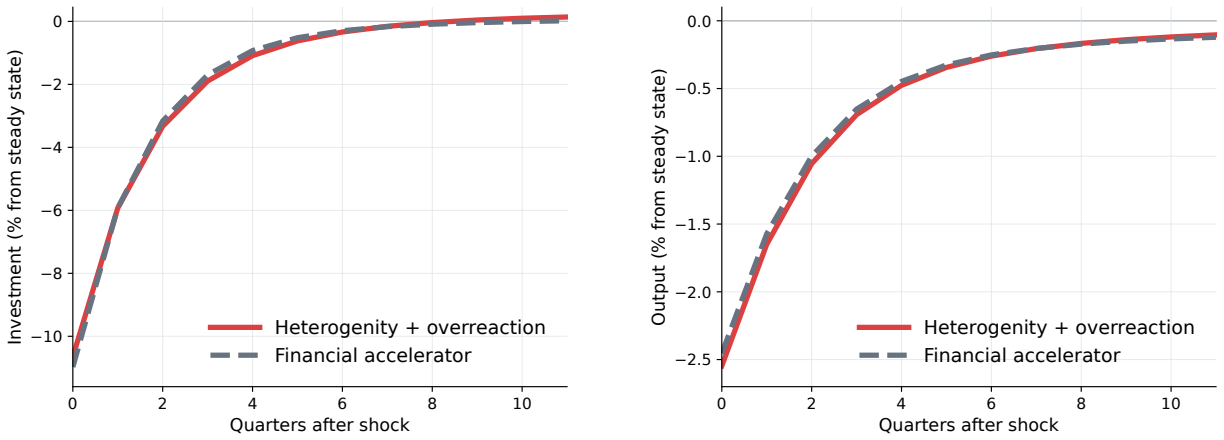
We now examine the transmission of aggregate shocks in our calibrated economy.

We solve for the non-FIRE equilibrium in Definition 2 using nonlinear MIT shocks (Boppart et al., 2018, Auclert et al., 2021). Given an initial level of aggregate productivity and an initial distribution of intermediary net worth, we compute the transition to an unanticipated aggregate shock, assuming that households and intermediaries do not anticipate further aggregate shocks. At each date, intermediaries form aggregate productivity forecasts using the expectation formation process in (29) and given these, use the FIRE equilibrium mapping to

[42] As already discussed in Section 2, throughout the paper we maintain the assumption that households form expectations according to FIRE.

[43] Alternatively, we could calibrate χ and η by matching the empirical target to the model-implied forecast-error response of an endogenous variable, such as the excess return μ_t. For a given empirical target, this approach would yield a larger χ because pessimism about productivity also lowers realized excess returns, partly offsetting the error in intermediaries' return forecasts.

Figure 8 – Transmission of aggregate shocks



(a) Investment **(b)** Output

Notes: the figure plots impulse responses to an unanticipated negative 1% aggregate productivity shock with quarterly persistence of 0.75. Panels (a) and (b) show investment and output, respectively. The solid red line shows our calibrated heterogeneous-intermediary model, with overreaction of expectations calibrated to match the estimates from Figure 7. The dashed gray line shows the financial accelerator benchmark with a representative constrained intermediary. All transition paths are computed fully nonlinearly.

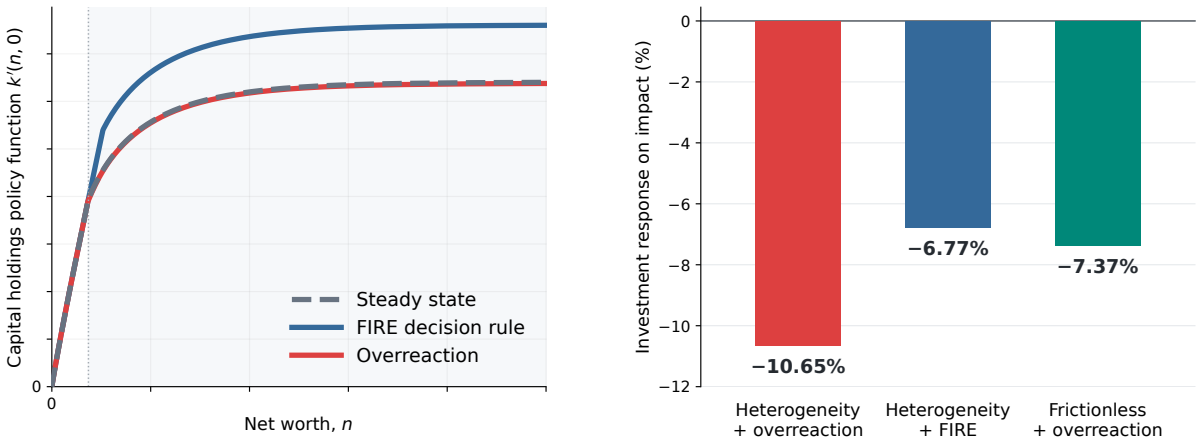
forecast future prices. Given these price forecasts we solve for intermediaries' value and policy functions backward, then use their choices and realized returns to propagate the net-worth distribution forward. For exercises where we simulate a sequence of shocks, we apply the same procedure as above and treat each innovation as unanticipated, taking the balance sheets and aggregate states inherited by the previous history as the initial conditions. This non-linear approach allows us to capture allows us to capture state dependence and nonlinear or asymmetric responses to shocks. However, we abstract from time-varying aggregate uncertainty and its effects on intermediaries' precautionary portfolio choices and risk premia.

Panels (a) and (b) of Figure 8 respectively plot the responses of investment and output to a negative aggregate productivity shock. We compare our calibrated heterogeneous-intermediary model with overreaction of expectations (solid red line) with the financial accelerator benchmark, in which the representative intermediary is constrained (dashed gray line).[44]

Although our calibrated economy features a mass of unconstrained intermediaries whose lending responds to expected returns, following a negative shock it generates aggregate dynamics in line with those of the financial accelerator benchmark, as investment and output display similar initial contractions and subsequent recoveries. Our model is thus able to recon-

[44]We calibrate the financial accelerator benchmark to the same aggregate steady state as our baseline, keeping the nonfinancial block and intermediary survival rate ϱ unchanged. We re-calibrate the financial friction parameter θ and the endowment of new entrants n^{entry} to match aggregate leverage, K/N and the aggregate dividend-to-output ratio.

Figure 9 – Inspecting the mechanism



(a) Policy functions **(b)** Decomposing amplification

Notes: panel (a) plots the policy functions for capital holdings $k'(n, 0)$ in steady state (dashed gray), and after a TFP shock under FIRE (solid blue) and with our calibrated overreaction of expectations (solid red). Panel (b) shows the on-impact investment response in our calibrated model with overreaction of expectations, with FIRE, and in a frictionless version of our model where expectations overreact as in our calibrated model.

ciles the forward-looking lending behavior we document in the microdata with the substantial amplification typical of the financial accelerator mechanism.

However, behind these similar aggregate dynamics lie very different underlying mechanisms. In the financial accelerator benchmark, the representative intermediary cuts lending because its financial constraint binds. In our model, the fall in capital prices also erodes constrained intermediaries' net worth and forces them to cut lending, but the extent to which this gives rise to amplification depends on the response of unconstrained intermediaries. Under FIRE, these intermediaries would expand lending in response to lower capital prices, partly offsetting the retrenchment of constrained intermediaries. When expectations overreact, pessimism about future returns instead leads them to cut lending as well, amplifying the contraction. Panel (a) of Figure 9 illustrates this mechanism by plotting intermediaries' lending policy functions $k'(n, 0)$ across net worth in the FIRE benchmark and when intermediaries' expectations overreact. In both cases, we use the realized price path from the economy with overreaction. The policy functions coincide at low levels of net worth, as the leverage constraint binds and intermediaries' expectations are irrelevant for their behavior. As net worth rises, the constraints loosen and lending becomes more sensitive to expected returns. Under FIRE, unconstrained intermediaries expand their capital holdings in anticipation of the recovery in capital prices. When expectations overreact, they expect lower returns and instead demand less capital.

In our model, the can't lend channel still plays an important role, as both overreaction of expectations and financial constraints are needed to generate substantial aggregate amplification while matching the microdata. To show this, panel (b) of Figure 9 compares the on-impact investment response in our calibrated economy with the response in the same model under FIRE and with that in an economy where expectations overreact but intermediaries face no financial constraints. The frictionless limit can match the forward-looking lending behavior documented in our microdata, but gives rise to an investment contraction about one-third smaller than in our heterogeneous-intermediary economy, even when expectations overreact. The financial accelerator limit generates large drops in output and investment, as shown in Figure 8, but cannot match our micro estimates because all intermediaries are constrained and their lending does not respond to their own expectations. Finally, replacing overreaction with FIRE in our heterogeneous-intermediary economy reduces the on-impact investment contraction by about 40%.

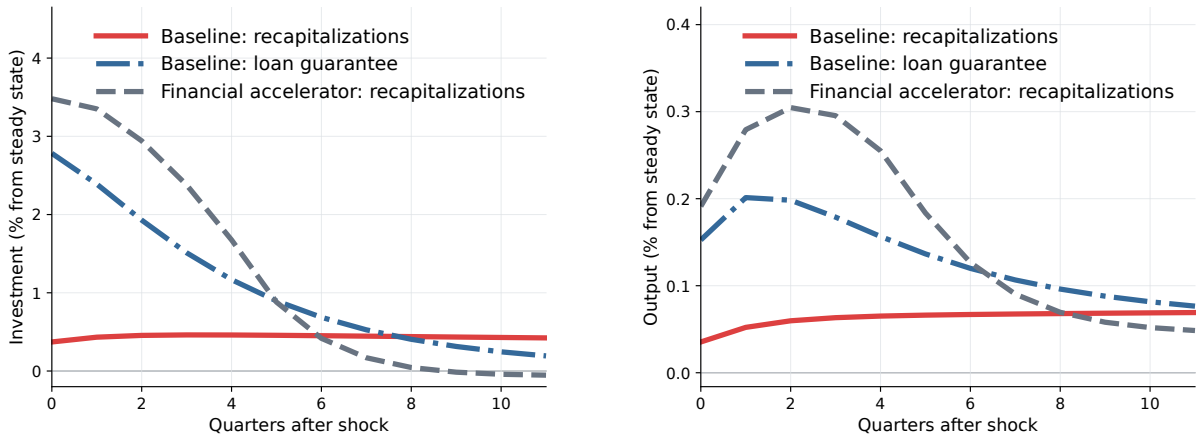
Together, overreaction of expectations and financial constraints allow our model to match the forward-looking lending behavior documented in our microdata while generating sizeable amplification consistent with the aggregate evidence on financial crises. Next, we show that while our framework gives rise to aggregate dynamics similar to those of the standard financial accelerator limit, matching the microdata leads to different predictions for the effectiveness of policy during crises.

5.3 Contrasting two policies

In the workhorse financial-accelerator model, investment collapses during recessions because intermediaries *cannot* lend: their balance-sheet constraints bind. In our model, by contrast, the marginal intermediary is unconstrained: it cuts back on lending not because it cannot, but because it *will not* lend. While Figure 8 establishes that the two models generate similar responses to aggregate shocks, we now show that the reason behind intermediaries' lending behavior has important implications for the design of policy interventions.

Consider two such policies. First, recapitalizations of the financial sector, the primary policy tool used during crises (Laeven and Valencia, 2026), which raise financial intermediaries' net worth, thereby expanding constrained intermediaries' capacity to lend. We model these as an increase in government transfers to intermediaries T_t^b. Second, loan guarantees protect intermediaries against unusually high default losses on new lending. The government covers a fraction $\tau \in [0, 1]$ of losses arising from realized default rates above their baseline level δ. Since

Figure 10 – Implications for policy: recapitalizations vs loan guarantees



(a) Investment **(b)** Output

Notes: the panels plot the investment and output responses to two policies: recapitalizations of financial intermediaries and loan guarantees. Both policies are normalized so that the net present value cost equals 1% of annual GDP. We model recapitalizations as an increase in government transfers to intermediaries, T_t^b. Under loan guarantees, the government covers a fraction τ of losses on new lending arising from default rates above their baseline level δ.

positive realizations of ε_{bt+1} increase defaults, the effective excess return is

$$\widehat{\mu}_{bt+1} = \frac{\left[1 - \delta - \varepsilon_{bt+1} + \tau \max\{\varepsilon_{bt+1}, 0\}\right] \left(r^k_{t+1} + q_{t+1}\right)}{q_t} - R_{t+1}.$$

Loan guarantees thus dampen adverse default realizations, while leaving favorable realizations unchanged. By scaling down losses in the left tail of the return distribution, they raise the expected return on new lending, so that

$$\mathbb{E}_t \widehat{\mu}_{bt+1} = \mathbb{E}_t \mu_{bt+1} + \frac{\tau \sigma_\varepsilon}{q_t \sqrt{2\pi}}.$$

By raising expected returns and reducing downside risk, loan guarantees therefore operate through expected returns, the margin unconstrained intermediaries are more responsive to.

Figure 10 plots the investment and output responses to the two policies. We scale the interventions to have the same ex-ante fiscal cost, measured as the expected present discounted value of government outlays.

In the financial accelerator limit, intermediaries are constrained and have a high marginal propensity to lend out of net worth. As a result, recapitalizations have a strong stimulative effect on investment and output. That is not the case in our calibrated model. Recapitalizations increase investment by constrained intermediaries, but the resulting fall increase in the price of

capital q_t and fall in future excess returns induces unconstrained intermediaries to reduce their capital holdings. In general equilibrium, these two forces largely offset each other, dampening the stimulative effects of recapitalizations: the investment and output responses are only one-tenth and one-fifth as large as in the financial accelerator limit, respectively.

Because loan guarantees directly raise expected returns on lending, the margin to which unconstrained intermediaries respond, they are substantially more effective at stimulating investment and output in our model. As Figure 10 shows, the on-impact responses of investment and output are roughly seven and four times as large as under recapitalizations, respectively. These results illustrate the importance of matching the moments we estimate in the microdata: while our model and the financial accelerator limit generate similar responses to aggregate shocks (Figure 8), they have different implications for the effectiveness of policy interventions (Figure 10).

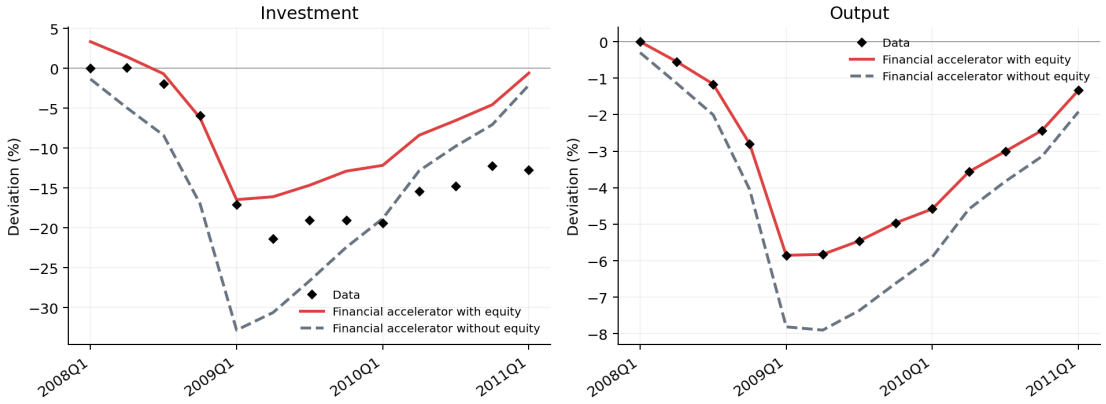
Revisiting the Global Financial Crisis. We now illustrate the implications of our results through a historical counterfactual of the euro-area economy during the Global Financial Crisis (GFC). To do so, we feed both our calibrated heterogeneous-intermediary model and the financial accelerator benchmark the path of recapitalizations observed over 2008q1–2011q1, totaling roughly 6% of GDP (Igan et al., 2019). For each model, we then back out the innovations to aggregate productivity that allow the model to match euro-area output given the recapitalizations.[45] We solve the model fully non-linearly and treat each innovation as an unanticipated MIT shock. Figures D.3 and D.4 in Appendix D report the path of aggregate productivity and recapitalizations. The corresponding paths for a broader set of macroeconomic and financial variables are reported in Figures D.5 to D.7 in Appendix D.5.

Through the lens of the standard financial accelerator model, recapitalizations played a key role in preventing a much deeper recession. Panel (a) of Figure 11 plots the implied paths of investment and output in this economy, in the left and right columns, respectively. By construction, the model matches the observed path of GDP when recapitalizations are in place. In the spirit of Orchard et al. (2025), we then ask how the recession would have unfolded without them. Removing the recapitalizations nearly doubles the investment contraction, which reaches 33% at the 2009q1 trough, and increases the output contraction by more than two percentage points.

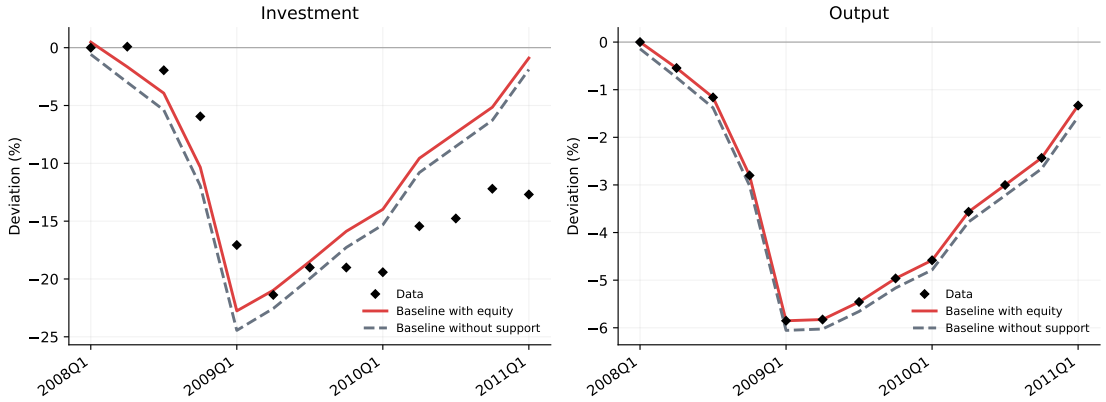
However, recapitalizations play a much smaller role in our calibrated heterogeneous-intermediary model, depicted in panel (b) of Figure 11. As before, the model matches the

[45] We match percentage deviations of model output from its pre-crisis steady state to those of real euro-area GDP from its 2008Q1 level. The GDP data come from Eurostat quarterly national accounts for 2008Q1–2011Q1 and are not detrended.

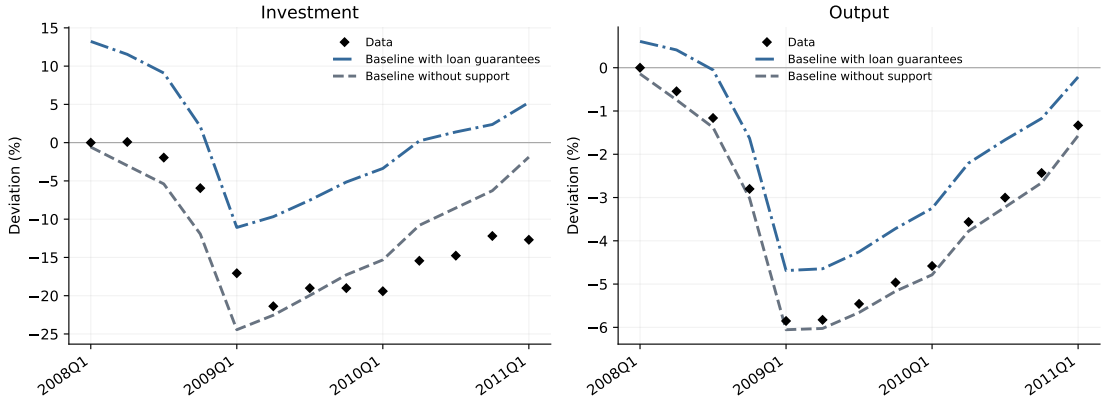
Figure 11 – Revisiting the Global Financial Crisis



(a) Financial accelerator: recapitalizations



(b) Baseline model: recapitalizations



(c) Baseline model: loan guarantees

Notes: the left and right columns show the paths of investment and output over 2008Q1–2011Q1. Panels (a) and (b) show the financial accelerator benchmark and our calibrated heterogeneous-intermediary model, respectively. Solid red lines show outcomes with recapitalizations; dashed gray lines show outcomes without. Panel (c) replaces recapitalizations with loan guarantees in the heterogeneous-intermediary model at the same discounted fiscal cost (blue dash-dotted lines). For each model, we recover productivity innovations to match observed GDP with recapitalizations and hold these innovations fixed across policy counterfactuals. Investment is not targeted. We solve the model fully nonlinearly. Black diamonds denote euro-area data from Eurostat quarterly national accounts. The data are expressed as percentage deviations from their 2008Q1 levels, without detrending; model outcomes are expressed as percentage deviations from steady state.

observed path of output by construction. In the presence of heterogeneity, the marginal intermediary remains relatively unconstrained throughout the recession. Recapitalizations therefore have small stimulative effects, reducing the investment contraction by 10% and the output contraction by 4%.[46]

We next consider replacing recapitalizations with loan guarantees at the same fiscal cost for the government, as shown in panel (c) of Figure 11. Since the marginal intermediary in the economy remains unconstrained throughout the crisis, loan guarantees are much more effective at stimulating the economy, in line with the results from Figure 10. Relatively to recapitalizations, they reduce the investment contraction by 8 percentage points and the output drop by 1 percentage point.

Taken together, these results show that while our model generates dynamics of aggregate output and investment during the GFC similar to those of the financial accelerator benchmark, matching the microdata leads to different conclusions about the effectiveness of crisis interventions.

Balance sheets, expectations, and recapitalizations in the data. Our model predicts that recapitalizations are more effective at stimulating output and investment in recessions where balance sheets, rather than expectations, are the main driver of bank lending.

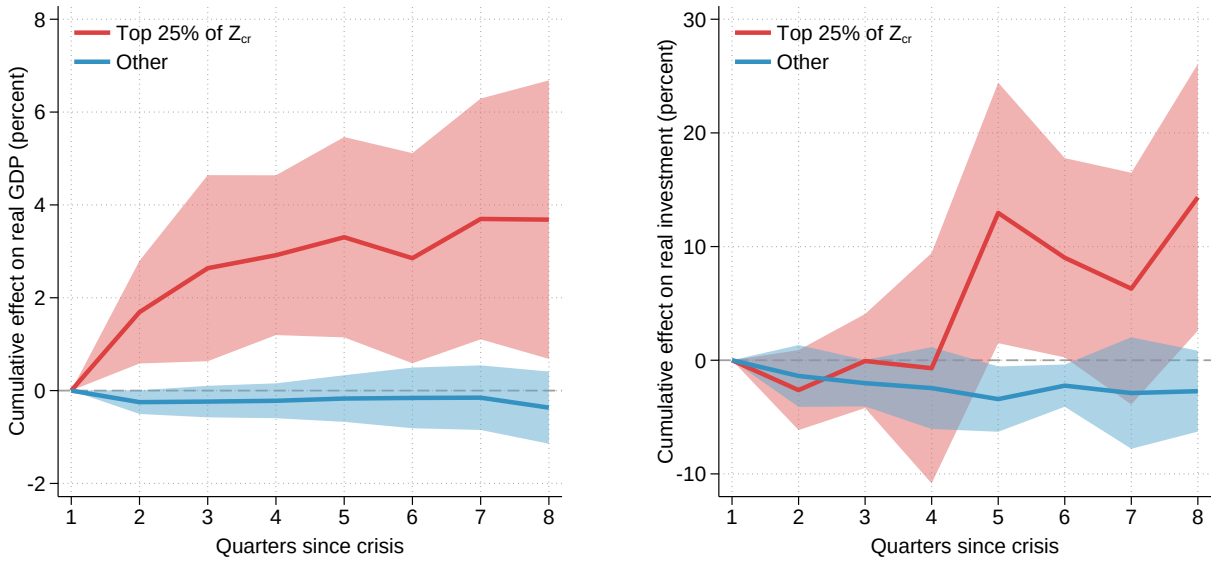
Figure 12 offers descriptive evidence consistent with this prediction. We obtain data covering 16 European countries across two episodes: the 2008 Global Financial Crisis and the 2012 euro-area sovereign debt crisis. For each country c and each episode r, we use the European Central Bank's Bank Lending Survey to measure the relative importance banks assigned to balance-sheet constraints and their expectations about the macroeconomic outlook when tightening credit standards. In particular, we define Z_{cr} as the ratio of the share of banks citing balance-sheet constraints as a factor behind tighter credit standards to the share citing the macroeconomic outlook:

$$Z_{cr} \equiv \frac{\%\text{ banks citing balance sheets}_{cr}}{\%\text{ banks citing macro outlook}_{cr}}.$$

We define D_{cr} as an indicator equal to one if country c is in the top quartile of the Z_{cr} distribution within episode r and zero otherwise. To examine whether the relationship between recapitalizations and the subsequent recovery differs across these groups, we estimate the fol-

[46]The decline in net worth during the recession tightens the marginal intermediary's balance-sheet constraint, so that recapitalizations are still more effective than starting from the steady-state distribution.

Figure 12 – Balance sheet, expectations, and recapitalizations in the data



(a) Real GDP **(b)** Real investment

Notes: the blue and red lines plot respectively $\widehat{\gamma}^h$ and $\widehat{\gamma}^h + \widehat{\beta}^h$ estimated from (30). The left and right panels use real GDP and real investment as dependent variables, respectively. Shaded areas represent 90% confidence intervals, constructed with heteroskedasticity-robust standard errors.

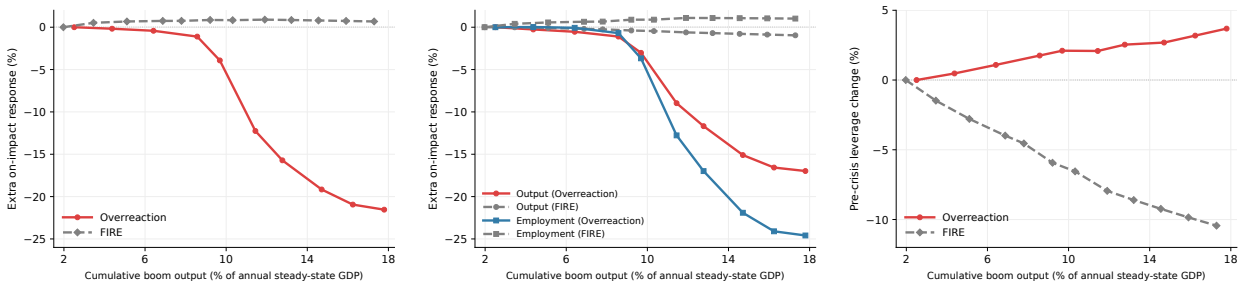
lowing local projection:

$$\Delta \ln Y_{cr+h} = \delta_r^h + \beta^h Recap_{cr} \times D_{cr} + \gamma^h Recap_{cr} + \chi^h D_{cr} + u_{crh}, \tag{30}$$

where $\Delta \ln Y_{cr+h} \equiv \ln Y_{cr+h} - \ln Y_{cr-1}$ is the cumulative log change in variable Y, δ_r^h is an episode fixed effect, and $Recap_{cr}$ denotes the total size of government recapitalizations of the financial sector in country c during episode r, as a share of GDP and standardized to have a standard deviation of one.

Panels (a) and (b) of Figure 12 plot the cumulative changes in real GDP and real investment, respectively, associated with a one-standard-deviation increase in recapitalizations for the top-quartile group ($\widehat{\gamma}^h + \widehat{\beta}^h$) and for the remaining observations ($\widehat{\gamma}^h$). Consistent with our model's prediction, we find that larger recapitalizations are associated with larger and faster recoveries of GDP and investment only in country-episodes where banks cited balance-sheet constraints relatively more often, suggesting that the marginal intermediary was constrained. Eight quarters after the crisis, a one-standard-deviation increase in recapitalizations (corresponding to about 4 percentage points of GDP) is associated with 3.8% higher GDP and 15.3% higher investment in this group. Instead, we find no statistically significant correlation in the remaining

Figure 13 – Expectations and the buildup of fragility during booms



(a) Investment **(b)** Output and employment **(c)** Leverage

Notes: we simulate sequences of positive 0.5% aggregate productivity shocks lasting between 2 and 24 quarters, each followed by the same negative 1% productivity shock. The horizontal axis measures cumulative output gains during the preceding expansion as a percentage of annual steady-state GDP. Panels (a) and (b) report the additional on-impact responses of investment and output and employment, respectively, relative to their responses to the same negative shock from steady state. Negative values indicate a larger contraction. Panel (c) plots the change in aggregate leverage of the financial sector in the period before the negative shock. We solve the model non-linearly with repeated unanticipated MIT shocks.

country-episodes, where banks cited the macroeconomic outlook relatively more often.

Our results thus carry an important lesson for policymakers. The potency of standard crisis-response tools such as recapitalizations depends on what pins down the marginal intermediary's behavior: its balance sheet or its expectations of future returns. This makes measures of intermediaries' beliefs, such as those from bank lending surveys, a valuable input into the design of crisis policy.

5.4 Fragility build up before crises

Finally, we show that matching the microdata also leads to different predictions for the buildup of fragility that makes recessions during financial crises so severe. A longstanding view, dating back to Minsky (1977) and Kindleberger (1978) argues that booms sow the seeds of the subsequent bust: fragility builds up in good times, leaving the economy more vulnerable to adverse shocks.

To study this idea in our setting, we feed the model progressively longer sequences of positive 0.5% aggregate productivity shocks, ranging from 2 to 24 quarters and generating increasingly large expansions. At the end of each expansion, we hit the economy with the same negative 1% aggregate productivity shock. Panels (a) and (b) of Figure 13 plot the additional declines in investment and output and emplyoment, respectively, induced by the negative shock after each expansion, relative to the decline the same shock would generate starting from steady state, against the preceding increase in output. We find that the larger the preceding expansion,

the more severe the recession induced by the same negative shock.

Our model thus provides a mechanism linking larger economic expansions to more severe subsequent recessions, helping rationalize the boom-bust patterns documented in the data (Jordà et al., 2013, Mian et al., 2017, 2020, Bordalo et al., 2024). As panel (c) of Figure 13 shows, overreaction makes intermediaries optimistic about future returns during the boom, leading them to increase leverage. When the negative shock hits, higher leverage amplifies the downturn through two channels. First, constrained intermediaries suffer larger losses in net worth, forcing them to contract lending more sharply. Second, unconstrained intermediaries become pessimistic about future productivity following the negative shock. Because they anticipate that a highly leveraged economy responds more strongly to bad realizations, they revise expected returns down by more and reduce lending further. Overreaction of expectations and balance-sheet constraints therefore interact to amplify the downturn. Appendix Figure D.8 separates the contributions of expectation overreaction, balance sheets, and their interaction to the additional contraction after the boom. The interaction between balance sheets and expectations accounts for one fifth of the additional output contraction, with the increasing in leverage accounting for one fourth and the reversal of optimism accounting for the remaining half. Balance sheets therefore amplify the downturn both directly, through larger losses in net worth, and indirectly, by amplifying the effect of pessimism on expected returns.

Matching the overreaction of expectations we estimate in the microdata is crucial for this result. As Figure 13 shows, under FIRE larger expansions are followed by milder contractions: high returns during the boom strengthen intermediaries' balance sheets, leaving them better able to absorb subsequent negative shocks.

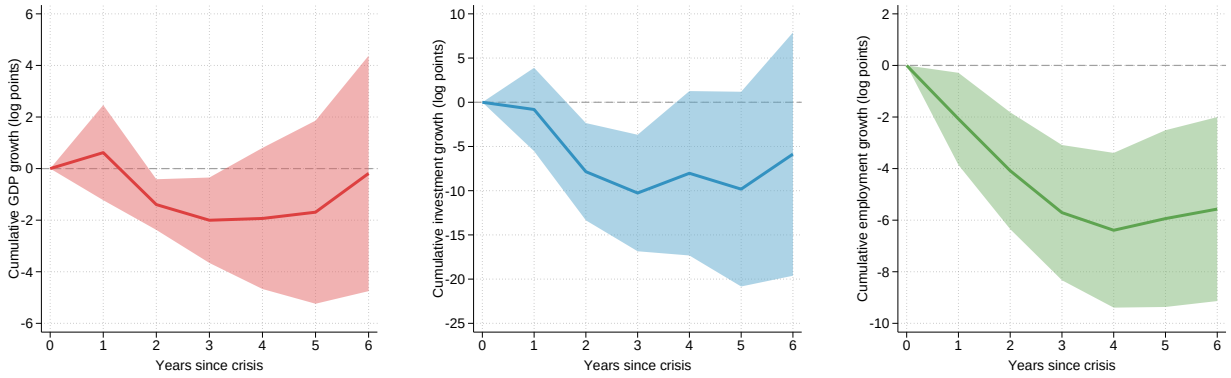
Expectations and crisis severity in the data. In our model, overreaction of expectations links optimism during the boom to the buildup of fragility. The model therefore predicts that crises preceded by greater optimism are more severe.

Figure 14 offers descriptive evidence consistent with this prediction. We combine GDP growth forecasts from the IMF's World Economic Outlook (WEO), available from 1990 onward, with the crisis chronology of Jordà et al. (2017). Our sample thus covers 18 crises across 15 countries. For each horizon h, we then estimate the following local projection:

$$\Delta \ln Y_{cr+h} = \alpha^h + \beta^h \Delta \log GDP^e_{cr} + \gamma^h \Delta \log GDP^{pre}_{cr} + u_{cr+h}, \tag{31}$$

where $\Delta \ln Y_{cr+h} \equiv \ln Y_{cr+h} - \ln Y_{cr-1}$ is the cumulative log change in variable Y and α^h is a constant. For each country c and crisis episode r, $\Delta \log GDP^e_{cr}$ denotes the average of the three-,

Figure 14 – Pre-crisis optimism and crisis severity



(a) Real GDP **(b)** Real investment **(c)** Employment

Notes: estimated $\widehat{\gamma}^h$ from (31) on a sample of 18 countries across 15 developed countries. Crises are defined as in Jordà et al. (2017). The outcome variable are real GDP, real investment and employment in panels (a), (b), and (c), respectively. Shaded areas represent 90% confidence bands, based on robust standard errors.

four-, and five-year-ahead GDP growth forecasts published in the WEO in the year before the crisis. We control for realized GDP growth in that same year, denoted by $\Delta \log GDP_{cr}^{pre}$.

Panel (a) of Figure 14 shows that more optimistic forecasts in the year before the crisis are associated with larger output declines during the subsequent recession, even after controlling for realized pre-crisis GDP growth. This correlation is economically large: three years after the start of the crisis, a one-standard-deviation higher forecast is associated with a 1.3% larger cumulative decline in GDP, roughly three-quarters of the GDP decline in the average crisis in our sample. Panels (b) and (c) show that more optimistic forecasts are also associated with larger declines in investment and employment. These results are consistent with our model's prediction that greater optimism during the expansion is associated with a more severe subsequent recession.

The results in this section show how the combination of overreaction of expectations and financial constraints allows our model to match the forward-looking lending behavior we document in the microdata while generating substantial amplification. Although our model generates aggregate dynamics similar to those of the financial accelerator benchmark, matching the microdata leads to different conclusions about why financial crises give rise to such large recessions and what policy can do about them. When the marginal intermediary is unconstrained, recapitalizations are less effective at stimulating the economy, while loan guarantees provide more stimulus at the same fiscal cost. Expectations also matter for the buildup of fragility before crises: optimism during expansions leads intermediaries to increase leverage, helping explain why larger economic booms are followed by more severe recessions.

6 Conclusion

Distinguishing between financial intermediaries' capacity and willingness to lend is crucial for understanding the effectiveness of stabilization policy during financial crises. We develop a model with heterogeneous financial intermediaries in which lending is governed by balance sheets for constrained intermediaries and by expected returns for those with room to lend. Whether unconstrained intermediaries offset constrained lenders' retrenchment or reinforce it depends on two moments: how strongly lending responds to expected returns and how expectations respond to aggregate shocks. Using 40 million bank forecasts of borrower default probabilities, we find that a one-percentage-point increase in a bank's expected portfolio return raises its total lending by 12%. We also find that expectations overreact to aggregate shocks, leaving banks excessively pessimistic following adverse news.

Once matched to these moments, the model generates substantial amplification and aggregate crisis dynamics similar to those of the standard financial accelerator, yet has sharply different policy implications. In a crisis, the marginal intermediary is unconstrained and cuts lending because it expects low returns. Recapitalizations expand lending by constrained intermediaries, but this additional lending raises the price of capital and lowers expected returns. Pessimistic unconstrained intermediaries cut back, largely offsetting the stimulus. Loan guarantees instead raise expected returns directly and stimulate output four times as much at the same expected fiscal cost. Substantial financial amplification therefore need not imply that recapitalizations are a powerful stabilization tool.

Several avenues for future research remain open. First, our findings raise normative questions about the design of stabilization policy. While we show that loan guarantees can support economic activity by offsetting the effects of pessimism on lending, our analysis abstracts from their potential to encourage excessive risk-taking. Assessing this trade-off would help determine when and how such interventions improve welfare. Second, our evidence motivates studying whether financial intermediaries' optimism and pessimism spread to borrowers' own beliefs through lending relationships, opening a new channel through which intermediaries' expectations propagate to the real economy.

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Can't Lend or Won't Lend? Beliefs, Balance Sheets and Financial Crises

Online Appendix

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A Appendix to Section 2

A.1 Derivation of the Autarky Value Function

We assume that upon diversion, intermediaries enter autarky with wealth $x = \theta q_t k'$. In autarky, they can consume and can only save in the risk-free asset, but cannot borrow or finance productive investment. Savings earn the steady-state return $\bar{R} = 1/\beta^{\text{HH}}$. Upon exogenous exit, they consumes their remaining wealth, with terminal utility $u(a')$.[47]

Let a_j denote wealth j periods after diversion, conditional on survival, and c_j denote consumption. The autarky value solves

$$V^\star(x) = \max_{\{c_j, a_{j+1}\}_{j\geq 0}} \sum_{j=0}^{\infty} (\beta\varrho)^j \left[u(c_j) + \beta(1-\varrho)u(a_{j+1})\right],$$

subject to

$$a_0 = x, \qquad a_{j+1} = \bar{R}(a_j - c_j), \qquad 0 \leq c_j \leq a_j.$$

In practice, we cast this problem recursively as

$$V^\star(x) = \max_{0\leq c\leq x} \left\{u(c) + \beta\left[\varrho V^\star(\bar{R}(x-c)) + (1-\varrho)u(\bar{R}(x-c))\right]\right\}.$$

To compute the outside option $V^\star(\theta q_t k')$ entering the incentive constraint in (4), we solve this Bellman equation numerically and evaluate it at different candidate policies.

A.2 Details on the Two-Intermediary Model

Constrained intermediary. Constrained intermediaries solve the same problem as in (4), in the limit in which $\gamma \to 0$ and $\underline{d} \to 0$:

$$\begin{aligned} V_t^c(n) &= \max_{d\geq 0, k'} \; d + \beta \widetilde{\mathbb{E}}_t \left[(1-\varrho)n' + \varrho V_{t+1}^c(n')\right] \\ s.t. \quad & n' = R_{t+1}(n-d) + \mu_{t+1} q_t k', \qquad \theta q_t k' \leq \mathbb{E}_t \, V_{t+1}^c(n). \end{aligned}$$

In this limit, utility is linear in dividends and the problem in (4) boils down to the original Gertler and Kiyotaki (2010), Gertler and Karadi (2011), so that idiosyncratic risk does not matter for either individual choices or aggregate dynamics.

[47] To distinguish intermediaries that have diverted from active intermediaries in the main text, in what follows we denote their wealth by a rather than n.

Both $\mathbf{\Theta}_B^c$ and $\mathbf{\Theta}_F^c$ are then functions of parameters only. In particular

$$\mathbf{\Theta}_B^c = \frac{A\,\bar{\eta}\bar{K}^c}{\bar{R}}\,\varrho(\bar{R}+\bar{\mu}\bar{\eta})\,\boldsymbol{D}\,(\boldsymbol{I}-\varrho(\bar{R}+\bar{\mu}\bar{\eta})\boldsymbol{L})^{-1}$$

and

$$\mathbf{\Theta}_F^c = \frac{A\,\bar{\eta}\bar{K}^c}{\bar{R}}\,\boldsymbol{D}\,(\boldsymbol{I}-\frac{\beta\varrho(\bar{R}+\bar{\mu}\bar{\eta})^2}{\bar{R}}\boldsymbol{F})^{-1}\boldsymbol{F},$$

where $\boldsymbol{F}$ is the one-period lead, and $A \equiv (1-\beta(\varrho(\bar{R}+\bar{\mu}\bar{\eta}))^2)/(1-\varrho(\bar{R}+\bar{\mu}\bar{\eta})\beta\varrho(\bar{R}+\bar{\mu}\bar{\eta})^2/\bar{R})$

Unconstrained intermediary. Unconstrained intermediaries solve the same problem as in (4) with CARA utility and taking the limit in which financial frictions disappear, i.e., $\underline{d} \to -\infty$, $\theta \to 0$. To keep analytical tractability we further assume that upon exit unconstrained intermediaries receive zero continuation value. They solve the following problem:

$$\begin{aligned} V_t^u(n) &= \max_{d,k'}\; u(d) + \beta\varrho\,\mathbb{E}_t\,V_{t+1}^u(n') \\ n' &= R_{t+1}(n-d) + \mu_{t+1}q_t k'. \end{aligned}$$

Which is equivalent to

$$\begin{aligned} V_t^u(n) &= \max_{d,k'}\; \frac{1-e^{-\gamma d}}{\gamma} + \beta\varrho\,\mathbb{E}_t\,V_{t+1}^u(n') \\ n' &= R_{t+1}(n-d) + \left(r_{t+1}^k + (1-\delta+\varepsilon_{t+1})q_{t+1} - R_{t+1}q_t\right)k'. \end{aligned}$$

Given this, the optimal portfolio and dividend choices are

$$k'_{t+1} = \nu_t\,q_t\,\frac{\mathbb{E}_t\,\mu_{t+1}}{\sigma_\varepsilon^2}, \qquad d_t = \varsigma_t\,n + \Lambda_t, \qquad \nu_t = \frac{1}{\gamma\varsigma_{t+1}}, \tag{A.1}$$

where

$$\frac{1}{\varsigma_t} = 1 + \frac{1}{\varsigma_{t+1}R_{t+1}}, \qquad \Lambda_t = \frac{1-\varsigma_t}{\gamma}\Big[S_t - \ln(\beta\varrho) - \ln R_{t+1}\Big] + (1-\varsigma_t)\Lambda_{t+1}, \tag{A.2}$$

and $S_t \equiv \left(q_t\,\mathbb{E}_t\,\mu_{t+1}\right)^2/(2\sigma_\varepsilon^2)$. In steady state $\bar{\varsigma} = \bar{r}/\bar{R}$, so that $\bar{\nu} = \frac{1}{\gamma}\frac{\bar{R}}{\bar{R}-1}$ and $\bar{\Lambda} = \left[\bar{S} - \ln(\beta\varrho\bar{R})\right]/\left(\gamma(\bar{R}-1)\right)$. Aggregating, $D_t^u = \varsigma_t N_t^u + \Lambda_t$ with

$$N_{t+1}^u = \varrho\big[R_{t+1}(N_t^u - D_t^u) + q_t K_{t+1}^u \mu_{t+1}\big] + (1-\varrho)n^e, \tag{A.3}$$

so that $\bar{N}^u = \varrho\big(\bar{K}^u\bar{\mu} - \bar{R}\bar{\Lambda}\big)/(1-\varrho) + n^e$.

As in the case of constrained intermediaries, $\boldsymbol{\Theta}^u$ and $\boldsymbol{B}$ are then functions of parameters only. In particular,

$$\boldsymbol{\Theta}^u = \frac{1}{\gamma\sigma_\varepsilon^2}\frac{\bar{R}}{\bar{R}-1}\boldsymbol{D}\boldsymbol{F},$$

and

$$\boldsymbol{B} = \frac{\bar{K}^u}{\bar{R}^2}\boldsymbol{D}\boldsymbol{F}^2\left(\boldsymbol{I}-\bar{R}^{-1}\boldsymbol{F}\right)^{-1}.$$

A.3 Proof of Proposition 1

Normalizing the adjustment cost of capital good producers so that $\Psi_{II}(\bar{I},\bar{K}) = 1$ and $\Psi_{IK}(\bar{I},\bar{K}) = 0$, we have that:[48]

$$\mathrm{d}\boldsymbol{q} = \mathrm{d}\boldsymbol{\mathcal{I}}$$

Imposing market clearing in the market for capital goods:

$$\mathrm{d}\boldsymbol{\mathcal{I}} = \lambda\mathrm{d}\boldsymbol{\mathcal{I}}^u + (1-\lambda)\mathrm{d}\boldsymbol{\mathcal{I}}^c$$

Plugging in (14) and (16) from the main text in the equation above gives

$$\begin{aligned}\mathrm{d}\boldsymbol{\mathcal{I}} =& \lambda\left(\boldsymbol{\Theta}^u\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{\mu} + \bar{K}^u\boldsymbol{D}\,\mathrm{d}\boldsymbol{q} - \boldsymbol{B}\,\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{R}\right)\\ &+ (1-\lambda)\left(\boldsymbol{\Theta}_B^c\left(\mathrm{d}\boldsymbol{\mu} + \tfrac{1}{\bar{\eta}}\mathrm{d}\boldsymbol{R}\right) + \boldsymbol{\Theta}_F^c\left(\mathbb{E}\,\mathrm{d}\boldsymbol{\mu} + \tfrac{1}{\bar{\eta}}\,\mathbb{E}\,\mathrm{d}\boldsymbol{R}\right) - \bar{K}^c\boldsymbol{D}\,\mathrm{d}\boldsymbol{q}\right)\end{aligned} \tag{A.4}$$

Using our assumptions, we have that $\mathrm{d}\boldsymbol{R} = \boldsymbol{0}$ and $\mathrm{d}\boldsymbol{q} = \mathrm{d}\boldsymbol{\mathcal{I}}$, so that:

$$\mathrm{d}\boldsymbol{\mathcal{I}} = \lambda\left(\boldsymbol{\Theta}^u\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{\mu} + \bar{K}^u\boldsymbol{D}\,\mathrm{d}\boldsymbol{I}\right) + (1-\lambda)\left(\boldsymbol{\Theta}_B^c\mathrm{d}\boldsymbol{\mu} + \boldsymbol{\Theta}_F^c\,\mathbb{E}\,\mathrm{d}\boldsymbol{\mu} - \bar{K}^c\boldsymbol{D}\,\mathrm{d}\boldsymbol{I}\right) \tag{A.5}$$

Using the definition of μ_t:

$$\mathrm{d}\boldsymbol{\mu} = \mathrm{d}\boldsymbol{r}^k + (1-\delta)\mathrm{d}\boldsymbol{q} - \bar{R}^k\boldsymbol{L}\mathrm{d}\boldsymbol{q}$$

The rental rate on capital r_t^k is pinned down by the first order condition of final good producers

$$r_t^k = (1-\alpha)A_t\left(\frac{H_t}{K_t}\right)^{\alpha}$$

Moreover, notice that the first order condition of final good producer for the labor choice gives

$$w_t = \alpha A_t\left(\frac{K_t}{H_t}\right)^{1-\alpha}$$

[48] This can be achieved, for example, by having $\Psi(I_t,K_t) = \frac{\psi}{2\delta\bar{K}}\left(I_t-\delta\bar{K}\right)^2$, as in Del Negro et al. (2017), and imposing a parametric restriction on ψ.

Hence:

$$r_t^k = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} A_t^{\frac{1}{1-\alpha}} w_t^{-\frac{\alpha}{1-\alpha}} \tag{A.6}$$

Moreover, the household optimality condition for labor supply gives

$$\zeta H_t^{\varphi} = w_t.$$

Under perfectly elastic labor supply we thus have that $\mathrm{d}\boldsymbol{w} = \mathbf{0}$. Linearizing (A.6) and plugging this, plus imposing parametric restrictions on α and $\bar{A}$ gives

$$\mathrm{d}\boldsymbol{r}^k = \mathrm{d}\boldsymbol{A}$$

Plugging back in (A.5) gives

$$\begin{aligned}\mathrm{d}\boldsymbol{\mathcal{I}} =& \lambda \left(\boldsymbol{\Theta}^u \left(\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} + (1-\delta)\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{\mathcal{I}} - \bar{R}^k \boldsymbol{L} \widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{\mathcal{I}} \right) + \bar{K}^u \boldsymbol{D}\, \mathrm{d}\boldsymbol{I} \right) \\ &+ (1-\lambda) \Big[\boldsymbol{\Theta}_B^c \left(\mathrm{d}\boldsymbol{A} + (1-\delta)\mathrm{d}\boldsymbol{\mathcal{I}} - \bar{R}^k \boldsymbol{L} \mathrm{d}\boldsymbol{\mathcal{I}} \right) + \boldsymbol{\Theta}_F^c \left(\mathbb{E} d\boldsymbol{A} + (1-\delta)\, \mathbb{E}\, \mathrm{d}\boldsymbol{\mathcal{I}} - \bar{R}^k \boldsymbol{L}\, \mathbb{E}\, \mathrm{d}\boldsymbol{\mathcal{I}} \right) \\ &- \bar{K}^c \boldsymbol{D}\, \mathrm{d}\boldsymbol{I} \Big]\end{aligned} \tag{A.7}$$

Imposing FIRE $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{X} = \mathrm{d}\boldsymbol{X}$ and rearranging:

$$\begin{aligned}\mathrm{d}\boldsymbol{\mathcal{I}} =& \left(\lambda\boldsymbol{\Theta}^u + (1-\lambda)(\boldsymbol{\Theta}_B^c + \boldsymbol{\Theta}_F^c)\right) \mathrm{d}\boldsymbol{A} \\ &+ \left(\lambda\boldsymbol{\Theta}^u + (1-\lambda)(\boldsymbol{\Theta}_B^c + \boldsymbol{\Theta}_F^c)\right) \left((1-\delta)\boldsymbol{I} - \bar{R}^k \boldsymbol{L}\right) \mathrm{d}\boldsymbol{\mathcal{I}} \\ &+ \left(\lambda\bar{K}^u - (1-\lambda)\bar{K}^c\right) \boldsymbol{D}\mathrm{d}\boldsymbol{\mathcal{I}}\end{aligned} \tag{A.8}$$

which is (18) in Proposition 1.

A.4 Response to a Belief Shock

we now consider a pure belief shock. We assume that intermediaries hold arbitrary expectations $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}$ about the path of future productivity, while keeping the realized path fixed ($\mathrm{d}\boldsymbol{A} = \mathbf{0}$).[49] We also assume that, given their expectations about exogenous productivity, financial intermediaries use the model-consistent mapping between TFP and aggregate outcomes to form expectations about endogenous objects. Rearranging equation (18) in Proposition 1, we

[49] Throughout the paper, we maintain that depositors (households) form expectations under full information rational expectations.

denote this mapping by $\mathcal{J}(\lambda)$:

$$\mathcal{J}(\lambda) \equiv \left[\boldsymbol{I} - \left(\boldsymbol{\Theta}(\lambda)((1-\delta)\boldsymbol{I} - \bar{R}^k\boldsymbol{L}) + \boldsymbol{\mathcal{R}}(\lambda)\right)\right]^{-1}\boldsymbol{\Theta}(\lambda). \tag{A.9}$$

The following proposition provides a decomposition of the general equilibrium investment response to a shock to intermediaries expectations.

Corollary A.1. *Consider a shock to intermediaries' beliefs about future productivity* $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}$*, holding fixed the realized path. The general equilibrium investment response in a small open economy with perfectly elastic labor supply is given by:*

$$\mathrm{d}\boldsymbol{\mathcal{I}} = \underbrace{\boldsymbol{\mathcal{M}}(\lambda)}_{\substack{\text{GE feedback through} \\ \text{indirect effect}}} \times \Big(\underbrace{\lambda\boldsymbol{\Theta}^u}_{\substack{\text{response to expected} \\ \text{change in rental rate}}} + \underbrace{\lambda\boldsymbol{\Theta}^u(1-\delta)\mathcal{J}(\lambda)}_{\substack{\text{response to expected} \\ \text{change in price of capital}}} \Big)\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}, \tag{A.10}$$

where $\boldsymbol{\mathcal{M}}(\lambda) \equiv \left[\boldsymbol{I} - \left((1-\lambda)(1-\delta)(\boldsymbol{\Theta}_B^c + \boldsymbol{\Theta}_F^c) - \boldsymbol{\Theta}(\lambda)\bar{R}^k\boldsymbol{L} + \boldsymbol{\mathcal{R}}(\lambda)\right)\right]^{-1}$.

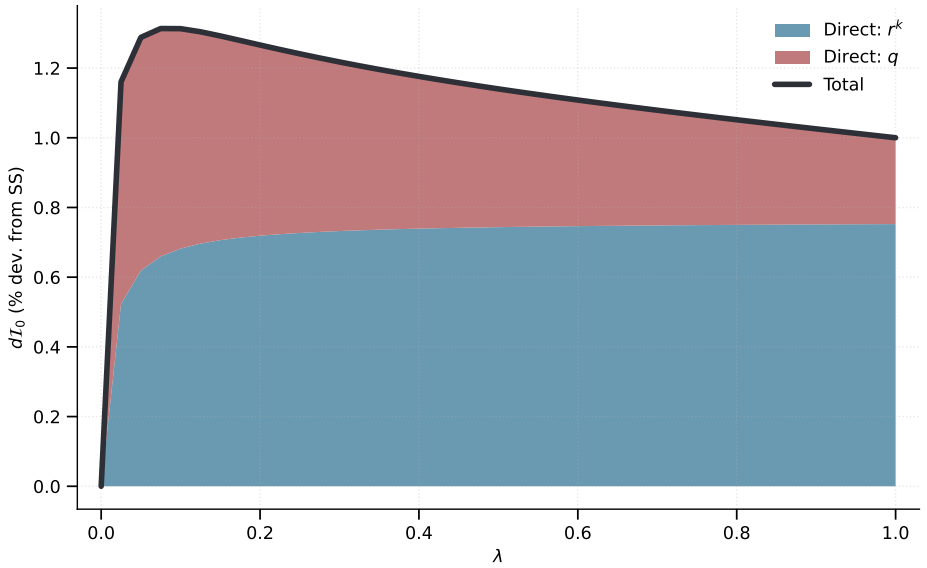
Proof. See Appendix A.5. □

Figure A.1 plots the on-impact investment response to a belief shock $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}$ as a function of the share of unconstrained intermediaries λ, together with the decomposition (A.10). The blue area represents the first term in the decomposition, which captures the investment response to intermediaries' expectations of the future rental rate of capital. Because only unconstrained intermediaries are responsive to expectations, this term is monotonically increasing in λ.

The red area instead plots the part of the response operating through intermediaries' expectations about the future price of capital, the second term in (A.10). Two opposing forces are at work here. A higher λ raises the mass of unconstrained intermediaries that respond to expectations, increasing the investment response. At the same time, a higher λ also dampens the equilibrium response of the price of capital to TFP, as Proposition 1 established, so that the same belief shock moves unconstrained intermediaries' expectations of future capital gains by less, decreasing the investment response.

As a result, while in partial equilibrium the investment response to beliefs is monotonically increasing in λ, in general equilibrium the relationship is non-monotone. Even though constrained intermediaries are unresponsive to expectations, a larger constrained share can make the economy *more* responsive to beliefs.

Figure A.1 – Response to expectation shock



Notes: the figure plots the on-impact investment response to a shock to intermediaries' expectations of future aggregate productivity $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}$, with quarterly persistence 0.5 as a function of the share of unconstrained intermediaries. The blue and red areas plot the decomposition characterized in Corollary A.1.

A.5 Proof of Corollary A.1

Let $\mathbb{E}\,\mathrm{d}\boldsymbol{A} = \mathbf{0}$, $\mathbb{E}\,\mathrm{d}\boldsymbol{\mathcal{I}} = \mathrm{d}\boldsymbol{\mathcal{I}}$, and $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{\mathcal{I}} = \boldsymbol{\mathcal{J}}(\lambda)\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}$. Under the assumption that unconstrained intermediaries observe the current price of capital, we can then plug this into (A.7) to obtain:

$$\begin{aligned}\mathrm{d}\boldsymbol{\mathcal{I}} =& \lambda\left(\boldsymbol{\Theta}^u + (1-\delta)\boldsymbol{\Theta}^u\boldsymbol{\mathcal{J}}(\lambda)\right)\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} - \lambda\boldsymbol{\Theta}^u\bar{R}^k\boldsymbol{L}\mathrm{d}\boldsymbol{\mathcal{I}} + \lambda\bar{K}^u\boldsymbol{D}\,\mathrm{d}\boldsymbol{I} \\ &+ (1-\lambda)\left[\boldsymbol{\Theta}^c_B\left((1-\delta)\mathrm{d}\boldsymbol{\mathcal{I}} - \bar{R}^k\boldsymbol{L}\mathrm{d}\boldsymbol{\mathcal{I}}\right) + \boldsymbol{\Theta}^c_F\left((1-\delta)\mathrm{d}\boldsymbol{\mathcal{I}} - \bar{R}^k\boldsymbol{L}\mathrm{d}\boldsymbol{\mathcal{I}}\right) - \bar{K}^c\boldsymbol{D}\,\mathrm{d}\boldsymbol{I}\right] \quad \text{(A.11)}\end{aligned}$$

Rearranging, we obtain (A.10) in Corollary A.1.

A.6 Proof of Proposition 2

In (A.7), let $\mathbb{E}\,\mathrm{d}\boldsymbol{A} = \mathrm{d}\boldsymbol{A}$, $\mathbb{E}\,\mathrm{d}\boldsymbol{\mathcal{I}} = \mathrm{d}\boldsymbol{\mathcal{I}}$, and $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{\mathcal{I}} = \boldsymbol{\mathcal{J}}(\lambda)\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}$. Under the assumption that unconstrained intermediaries observe the current price of capital, we can then write:

$$\begin{aligned}\mathrm{d}\boldsymbol{\mathcal{I}} =& \lambda\left(\boldsymbol{\Theta}^u\left(\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} + (1-\delta)\boldsymbol{\mathcal{J}}(\lambda)\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} - \bar{R}^k\boldsymbol{L}\mathrm{d}\boldsymbol{\mathcal{I}}\right) + \bar{K}^u\boldsymbol{D}\,\mathrm{d}\boldsymbol{\mathcal{I}}\right) \\ &+ (1-\lambda)\left[\boldsymbol{\Theta}^c_B\left(\mathrm{d}\boldsymbol{A} + (1-\delta)\mathrm{d}\boldsymbol{\mathcal{I}} - \bar{R}^k\boldsymbol{L}\mathrm{d}\boldsymbol{\mathcal{I}}\right) + \boldsymbol{\Theta}^c_F\left(\mathrm{d}\boldsymbol{A} + (1-\delta)\mathrm{d}\boldsymbol{\mathcal{I}} - \bar{R}^k\boldsymbol{L}\mathrm{d}\boldsymbol{\mathcal{I}}\right)\right. \\ &\left. - \bar{K}^c\boldsymbol{D}\,\mathrm{d}\boldsymbol{\mathcal{I}}\right] \quad \text{(A.12)}\end{aligned}$$

Define $\boldsymbol{Z} \equiv \mathrm{d}\boldsymbol{\mathcal{I}} - \mathrm{d}\boldsymbol{\mathcal{I}}^{\text{FIRE}}$ where $\mathrm{d}\boldsymbol{\mathcal{I}}^{\text{FIRE}}$ solves (A.12) under $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} = \mathrm{d}\boldsymbol{A}$. Then subtract $\mathrm{d}\boldsymbol{\mathcal{I}}^{\text{FIRE}}$ from both sides of (A.12):

$$\begin{aligned}\boldsymbol{Z} =& \lambda \left(\boldsymbol{\Theta}^u \left(\left(\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} - \mathrm{d}\boldsymbol{A} \right) + (1-\delta)\boldsymbol{\mathcal{J}}(\lambda) \left(\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} - \mathrm{d}\boldsymbol{A} \right) - \bar{R}^k \boldsymbol{L}\boldsymbol{Z} \right) + \bar{K}^u \boldsymbol{D}\,\boldsymbol{Z} \right) \\ &+ (1-\lambda) \left[\boldsymbol{\Theta}_B^c \left((1-\delta)\boldsymbol{Z} - \bar{R}^k \boldsymbol{L}\boldsymbol{Z} \right) + \boldsymbol{\Theta}_F^c \left((1-\delta)\boldsymbol{Z} - \bar{R}^k \boldsymbol{L}\boldsymbol{Z} \right) - \bar{K}^c \boldsymbol{D}\,\boldsymbol{Z} \right] \end{aligned} \tag{A.13}$$

Rearranging

$$\boldsymbol{Z} = \lambda \boldsymbol{\Theta}^u \left(\boldsymbol{I} + (1-\delta)\boldsymbol{\mathcal{J}}(\lambda) \right) \left(\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} - \mathrm{d}\boldsymbol{A} \right) + \boldsymbol{B}(\lambda)\boldsymbol{Z} \tag{A.14}$$

where

$$\boldsymbol{B}(\lambda) \equiv (1-\lambda)(1-\delta)(\boldsymbol{\Theta}_F^c + \boldsymbol{\Theta}_B^c) - \boldsymbol{\Theta}(\lambda)\bar{R}^k \boldsymbol{L} + \boldsymbol{\mathcal{R}}(\lambda) \tag{A.15}$$

Which gives

$$\boldsymbol{Z} = \left(\boldsymbol{I} - \boldsymbol{B}(\lambda) \right)^{-1} \lambda \boldsymbol{\Theta}^u \left(\boldsymbol{I} + (1-\delta)\boldsymbol{\mathcal{J}}(\lambda) \right) \left(\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} - \mathrm{d}\boldsymbol{A} \right) \tag{A.16}$$

Notice that

$$\left(\boldsymbol{I} - \boldsymbol{B}(\lambda) \right)^{-1} = \left(\boldsymbol{I} - (1-\lambda)(1-\delta)(\boldsymbol{\Theta}_F^c + \boldsymbol{\Theta}_B^c) + \boldsymbol{\Theta}(\lambda)\bar{R}^k \boldsymbol{L} - \boldsymbol{\mathcal{R}}(\lambda) \right)^{-1} = \boldsymbol{\mathcal{M}}(\lambda) \tag{A.17}$$

Thus, (A.16) becomes:

$$\boldsymbol{Z} = \boldsymbol{\mathcal{M}}(\lambda)\lambda \boldsymbol{\Theta}^u \left(\boldsymbol{I} + (1-\delta)\boldsymbol{\mathcal{J}}(\lambda) \right) \left(\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} - \mathrm{d}\boldsymbol{A} \right) = \widetilde{\boldsymbol{\mathcal{J}}}(\lambda) \left(\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} - \mathrm{d}\boldsymbol{A} \right) \tag{A.18}$$

Finally, using the definition of $\boldsymbol{Z}$ we obtain

$$\mathrm{d}\boldsymbol{\mathcal{I}} = \mathrm{d}\boldsymbol{\mathcal{I}}^{\text{FIRE}} + \widetilde{\boldsymbol{\mathcal{J}}}(\lambda) \left(\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} - \mathrm{d}\boldsymbol{A} \right) = \boldsymbol{\mathcal{J}}(\lambda)\mathrm{d}\boldsymbol{A} + \widetilde{\boldsymbol{\mathcal{J}}}(\lambda) \left(\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} - \mathrm{d}\boldsymbol{A} \right) \tag{A.19}$$

which is (19) in Proposition 2. Finally, to recover (20) in the main text, impose the expectation formation process $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} = \chi \mathrm{d}\boldsymbol{A}$ and substitute in (A.19).

A.7 Proposition 1: Relaxing the Assumptions

The following proposition extends Proposition 1 in the main text to the case where we relax the three simplifying assumptions in the main text. In particular, we dispense with the small

open economy assumption and let the risk-free rate be determined endogenously in equilibrium, assume standard separable preferences between hours worked and consumption for the household (and hence relax at the same time the assumption of GHH utility as well as the perfectly elastic labor supply assumption):

$$v(C_t, H_t) = \frac{C_t^{1-\gamma^{\text{HH}}}}{1-\gamma^{\text{HH}}} - \zeta \frac{H_t^{1+\varphi}}{1+\varphi}$$

Finally, we define the adjustment cost of capital good producers to follow a standard quadratic form:

$$\Psi(I_t, K_t) = \frac{\psi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$$

Proposition A.1. *The general-equilibrium investment response to a TFP shock under FIRE in a closed economy is*

$$\begin{pmatrix} \mathrm{d}\boldsymbol{\mathcal{I}} \\ \mathrm{d}\boldsymbol{D} \end{pmatrix} = \left[\boldsymbol{I} - \boldsymbol{\mathcal{G}}(\lambda)\right]^{-1} \boldsymbol{\mathcal{A}}(\lambda)\, \mathrm{d}\boldsymbol{A}, \tag{A.20}$$

where

$$\boldsymbol{\mathcal{A}}(\lambda) \equiv \frac{1+\varphi}{\widetilde{\phi}\bar{A}}\left(\bar{r}^k\, \boldsymbol{X}(\lambda) + \frac{\gamma^{HH}\bar{R}\bar{Y}}{\bar{C}}\, \boldsymbol{Z}(\lambda)\, \boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I})\right), \tag{A.21}$$

and

$$\begin{aligned}\boldsymbol{\mathcal{G}}(\lambda) \equiv & \left[\boldsymbol{X}(\lambda)((1-\delta)\boldsymbol{I} - \bar{R}^k\boldsymbol{L}) + \boldsymbol{Y}(\lambda)\right]\frac{\psi}{\bar{K}}\boldsymbol{D}^{-1}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{S} + \boldsymbol{X}(\lambda)\,\frac{\alpha\bar{r}^k}{\widetilde{\phi}}\left(\frac{\gamma^{HH}}{\bar{C}}\boldsymbol{J} - \frac{\varphi+\gamma^{HH}\frac{\bar{Y}}{\bar{C}}}{\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S}\right) \\ & + \boldsymbol{Z}(\lambda)\,\frac{\gamma^{HH}\bar{R}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I})}{\bar{C}\widetilde{\phi}}\left(\frac{(1+\varphi)(1-\alpha)\bar{Y}}{\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S} - (1+\varphi-\alpha)\boldsymbol{J}\right),\end{aligned}$$

with $\widetilde{\phi} \equiv 1+\varphi-\alpha+\alpha\gamma^{HH}\frac{\bar{Y}}{\bar{C}}$, $\bar{C} = \bar{Y} - \bar{\mathcal{I}} - \lambda\bar{D}^u - (1-\lambda)\bar{D}^c$, $\boldsymbol{S} \equiv (\boldsymbol{I}\ \ \boldsymbol{0})$, $\boldsymbol{J} \equiv (\boldsymbol{I}\ \ \boldsymbol{I})$, *and*

$$\begin{aligned}\boldsymbol{X}(\lambda) &\equiv \begin{pmatrix} \boldsymbol{\Theta}(\lambda) \\ \lambda\boldsymbol{\Gamma}\boldsymbol{F} + (1-\lambda)\boldsymbol{\Gamma}^c \end{pmatrix}, \\ \boldsymbol{Y}(\lambda) &\equiv \begin{pmatrix} (\lambda\bar{K}^u - (1-\lambda)\bar{K}^c)\boldsymbol{D} \\ \lambda\bar{\mu}\boldsymbol{\Gamma} \end{pmatrix}, \\ \boldsymbol{Z}(\lambda) &\equiv \begin{pmatrix} \frac{1-\lambda}{\bar{\eta}}(\boldsymbol{\Theta}_B^c + \boldsymbol{\Theta}_F^c) - \lambda\boldsymbol{B} - \boldsymbol{\Theta}(\lambda) \\ \lambda\boldsymbol{\Pi} - \lambda\boldsymbol{\Gamma}\boldsymbol{F} + (1-\lambda)\left(\frac{1}{\bar{\eta}}-1\right)\boldsymbol{\Gamma}^c \end{pmatrix},\end{aligned}$$

where, with $\boldsymbol{U} \equiv (\boldsymbol{I} - \bar{R}^{-1}\boldsymbol{F})^{-1}$, $\boldsymbol{N} \equiv \varrho(\boldsymbol{I} - \varrho\boldsymbol{L})^{-1}\boldsymbol{L}$, $\boldsymbol{G} \equiv (\boldsymbol{I} - \varrho\bar{R}^{\ell}\boldsymbol{L})^{-1}$ *and* $\boldsymbol{U}_{\phi} \equiv \left(\boldsymbol{I} - \frac{\beta\varrho(\bar{R}^{\ell})^2}{\bar{R}}\boldsymbol{F}\right)^{-1}\boldsymbol{F}$,

$$
\begin{aligned}
\boldsymbol{\Gamma}^{c} &\equiv (1-\varrho)\bar{K}^{c}\,\boldsymbol{G}\left[\boldsymbol{I} + \tfrac{\bar{\mu}\bar{\eta}}{\bar{R}}\boldsymbol{L}\boldsymbol{U}_{\phi}\right], \\
\boldsymbol{\Gamma} &\equiv \bar{\varsigma}\bar{K}^{u}\left[\boldsymbol{N}\left(2\boldsymbol{I} - \bar{\varsigma}\boldsymbol{U}\right) + \bar{R}^{-1}\boldsymbol{U}\right], \\
\boldsymbol{\Pi} &\equiv \left(\boldsymbol{I} - \bar{r}\boldsymbol{N}\right)\boldsymbol{U}\left[\tfrac{\bar{\varsigma}\bar{N}^{u} - \gamma^{-1}}{\bar{R}^{2}}\boldsymbol{I} - \tfrac{\bar{\varsigma}\bar{\Lambda}}{\bar{R}}\boldsymbol{U}\right]\boldsymbol{F} + \bar{\varsigma}\boldsymbol{N}\left[\left(\bar{N}^{u} - \bar{D}^{u}\right)\boldsymbol{F} - \bar{\mu}\,\boldsymbol{D}^{-1}\boldsymbol{B}\right].
\end{aligned}
$$

Proof. As the rental rate r_t^k now depends on the capital stock, it is useful to linearize the aggregate law of motion for capital and express

$$
\mathrm{d}\boldsymbol{K} = \boldsymbol{D}^{-1}\boldsymbol{L}\,\mathrm{d}\boldsymbol{\mathcal{I}} \tag{A.22}
$$

Moreover, the optimality condition of capital good producers gives:

$$
\mathrm{d}\boldsymbol{q} = \frac{\psi}{\bar{K}}\,\boldsymbol{D}^{-1}(\boldsymbol{I} - \boldsymbol{L})\,\mathrm{d}\boldsymbol{\mathcal{I}} \tag{A.23}
$$

The household optimal labor supply condition now is

$$
\zeta H_t^{\varphi} = C_t^{-\gamma^{\mathrm{HH}}} w_t
$$

Linearizing and combining with labor demand from final good producers gives

$$
(1+\varphi)\frac{dH_t}{\bar{H}} = \frac{dY_t}{\bar{Y}} - \gamma^{\mathrm{HH}}\frac{dC_t}{\bar{C}} \tag{A.24}
$$

Using the resource constraint and because investment adjustment costs are zero in steady state we have that

$$
\mathrm{d}C_t = \mathrm{d}Y_t - \mathrm{d}\mathcal{I}_t - \mathrm{d}D_t
$$

Substituting in the equation above the linearized production function as well as (A.24) gives

$$
\frac{\mathrm{d}Y_t}{\bar{Y}}\left(1 - \frac{\alpha}{1+\varphi} + \frac{\alpha\gamma^{\mathrm{HH}}}{1+\varphi}\frac{\bar{Y}}{\bar{C}}\right) = \frac{\mathrm{d}A_t}{\bar{A}} + \frac{\alpha\gamma^{\mathrm{HH}}}{1+\varphi}\frac{\mathrm{d}\mathcal{I}_t + \mathrm{d}D_t}{\bar{C}} + (1-\alpha)\frac{\mathrm{d}K_t}{\bar{K}} \tag{A.25}
$$

And plugging back in the resource constraint gives

$$
\mathrm{d}\boldsymbol{C} = \frac{(1+\varphi)\bar{Y}}{(1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}})\bar{A}}\mathrm{d}\boldsymbol{A} - \frac{1+\varphi-\alpha}{1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}}\mathrm{d}\boldsymbol{D}
$$

$$+\left(\frac{(1+\varphi)(1-\alpha)\bar{Y}}{(1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}})\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}-\frac{1+\varphi-\alpha}{1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}}\boldsymbol{I}\right)\mathrm{d}\boldsymbol{\mathcal{I}} \tag{A.26}$$

Linearizing the final good producer's optimality condition for the rental rate on capital and plugging in (A.25) gives:

$$\begin{aligned}\mathrm{d}\boldsymbol{r}^k=&\frac{(1+\varphi)\bar{r}^k}{(1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}})\bar{A}}\mathrm{d}\boldsymbol{A}+\frac{\alpha\gamma^{\mathrm{HH}}\bar{r}^k}{(1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}})\bar{C}}\mathrm{d}\boldsymbol{D}\\&+\left(\frac{\alpha\gamma^{\mathrm{HH}}\bar{r}^k}{(1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}})\bar{C}}\boldsymbol{I}-\frac{\alpha(\varphi+\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}})\bar{r}^k}{(1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}})\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\right)\mathrm{d}\boldsymbol{\mathcal{I}}\end{aligned} \tag{A.27}$$

The linearized Euler equation reads:

$$\mathrm{d}\boldsymbol{R}=\frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I})\mathrm{d}\boldsymbol{C} \tag{A.28}$$

Plugging in (A.26):

$$\begin{aligned}\mathrm{d}\boldsymbol{R}=&\frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I})\frac{(1+\varphi)\bar{Y}}{(1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}})\bar{A}}\mathrm{d}\boldsymbol{A}-\frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I})\frac{1+\varphi-\alpha}{1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}}\mathrm{d}\boldsymbol{D}\\&+\frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I})\left(\frac{(1+\varphi)(1-\alpha)\bar{Y}}{(1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}})\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}-\frac{1+\varphi-\alpha}{1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}}\boldsymbol{I}\right)\mathrm{d}\boldsymbol{\mathcal{I}}\end{aligned} \tag{A.29}$$

Aggregate dividends are $D_t=\lambda D_t^u+(1-\lambda)D_t^c$. For the unconstrained type, linearizing (A.1)–(A.3) and writing $\boldsymbol{U}\equiv(\boldsymbol{I}-\bar{R}^{-1}\boldsymbol{F})^{-1}$, $\boldsymbol{N}\equiv\varrho(\boldsymbol{I}-\varrho\boldsymbol{L})^{-1}\boldsymbol{L}$,

$$\mathrm{d}\boldsymbol{\varsigma}=\tfrac{\bar{\varsigma}}{\bar{R}^2}\boldsymbol{U}\boldsymbol{F}\,\mathrm{d}\boldsymbol{R},\qquad \mathrm{d}\boldsymbol{S}=\tfrac{\bar{\mu}}{\sigma_\varepsilon^2}\big(\boldsymbol{F}\,\mathrm{d}\boldsymbol{\mu}+\bar{\mu}\,\mathrm{d}\boldsymbol{q}\big),\qquad \mathrm{d}\boldsymbol{k}^{u\prime}=\tfrac{\bar{v}}{\sigma_\varepsilon^2}\boldsymbol{F}\,\mathrm{d}\boldsymbol{\mu}+\bar{K}^u\,\mathrm{d}\boldsymbol{q}-\boldsymbol{D}^{-1}\boldsymbol{B}\,\mathrm{d}\boldsymbol{R}, \tag{A.30}$$

$$\mathrm{d}\boldsymbol{\Lambda}=\boldsymbol{U}\Big[-\bar{R}\bar{\Lambda}\,\mathrm{d}\boldsymbol{\varsigma}+\tfrac{1}{\gamma\bar{R}}\mathrm{d}\boldsymbol{S}-\tfrac{1}{\gamma\bar{R}^2}\boldsymbol{F}\,\mathrm{d}\boldsymbol{R}\Big], \tag{A.31}$$

$$\mathrm{d}\boldsymbol{N}^u=\boldsymbol{N}\Big[-\bar{R}\big(\bar{N}^u\mathrm{d}\boldsymbol{\varsigma}+\mathrm{d}\boldsymbol{\Lambda}\big)+\big(\bar{N}^u-\bar{D}^u\big)\boldsymbol{F}\,\mathrm{d}\boldsymbol{R}+\bar{K}^u\bar{\mu}\,\mathrm{d}\boldsymbol{q}+\bar{\mu}\,\mathrm{d}\boldsymbol{k}^{u\prime}+\bar{K}^u\boldsymbol{F}\,\mathrm{d}\boldsymbol{\mu}\Big], \tag{A.32}$$

where we used $\bar{R}(1-\bar{\varsigma})=1$, so that $\mathrm{d}\boldsymbol{D}^u=\bar{\varsigma}\,\mathrm{d}\boldsymbol{N}^u+\bar{N}^u\mathrm{d}\boldsymbol{\varsigma}+\mathrm{d}\boldsymbol{\Lambda}=\boldsymbol{\Gamma}\big(\boldsymbol{F}\,\mathrm{d}\boldsymbol{\mu}+\bar{\mu}\,\mathrm{d}\boldsymbol{q}\big)+\boldsymbol{\Pi}\,\mathrm{d}\boldsymbol{R}$. For the constrained type, $\mathrm{d}\boldsymbol{D}^c=\frac{1-\varrho}{\varrho}\mathrm{d}\boldsymbol{N}^c$ with $\mathrm{d}\boldsymbol{N}^c=\varrho\bar{N}^c\boldsymbol{G}\big[\bar{\eta}\,\mathrm{d}\boldsymbol{\mu}+\mathrm{d}\boldsymbol{R}+\bar{\mu}\boldsymbol{L}\,\mathrm{d}\boldsymbol{\eta}\big]$ and $\mathrm{d}\boldsymbol{\eta}=\frac{\bar{\eta}^2}{\bar{R}}\boldsymbol{U}_\phi\big(\mathrm{d}\boldsymbol{\mu}+\frac{1}{\bar{\eta}}\mathrm{d}\boldsymbol{R}\big)$, so that, using $\bar{\eta}\bar{N}^c=\bar{K}^c$, $\mathrm{d}\boldsymbol{D}^c=\boldsymbol{\Gamma}^c\big(\mathrm{d}\boldsymbol{\mu}+\frac{1}{\bar{\eta}}\mathrm{d}\boldsymbol{R}\big)$. Collecting,

$$\mathrm{d}\boldsymbol{D}=\lambda\Big[\boldsymbol{\Gamma}\big(\boldsymbol{F}\,\mathrm{d}\boldsymbol{\mu}+\bar{\mu}\,\mathrm{d}\boldsymbol{q}\big)+\boldsymbol{\Pi}\,\mathrm{d}\boldsymbol{R}\Big]+(1-\lambda)\,\boldsymbol{\Gamma}^c\Big(\mathrm{d}\boldsymbol{\mu}+\tfrac{1}{\bar{\eta}}\mathrm{d}\boldsymbol{R}\Big), \tag{A.33}$$

with $\boldsymbol{\Gamma}$, $\boldsymbol{\Pi}$ and $\boldsymbol{\Gamma}^c$ as in the proposition.

Remember the expression for aggregate investment

$$\mathrm{d}\boldsymbol{\mathcal{I}} = \boldsymbol{\Theta}(\lambda)\mathrm{d}\boldsymbol{\mu} + (\lambda\bar{K}^u - (1-\lambda)\bar{K}^c)\boldsymbol{D}\mathrm{d}\boldsymbol{q} + \left(\tfrac{1-\lambda}{\bar{\eta}}\left(\boldsymbol{\Theta}_B^c + \boldsymbol{\Theta}_F^c\right) - \lambda\boldsymbol{B}\right)\mathrm{d}\boldsymbol{R} \tag{A.34}$$

Finally, substituting the definition of the excess return, as well as (A.23), (A.27), and (A.29) gives:

$$\begin{aligned}
\mathrm{d}\boldsymbol{\mathcal{I}} = &\left[\frac{(1+\varphi)\bar{r}^k}{\left(1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}\right)\bar{A}}\boldsymbol{\Theta}(\lambda) + \left(-\boldsymbol{\Theta}(\lambda) + \frac{1-\lambda}{\bar{\eta}}\left(\boldsymbol{\Theta}_B^c + \boldsymbol{\Theta}_F^c\right) - \lambda\boldsymbol{B}\right)\frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I})\right.\\
&\left.\times\frac{(1+\varphi)\bar{Y}}{\left(1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}\right)\bar{A}}\right]\mathrm{d}\boldsymbol{A}\\
&+\left\{\boldsymbol{\Theta}(\lambda)\left[\frac{\alpha\gamma^{\mathrm{HH}}\bar{r}^k}{\left(1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}\right)\bar{C}}\boldsymbol{I} - \frac{\alpha\left(\varphi+\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}\right)\bar{r}^k}{\left(1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}\right)\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\right.\right.\\
&\left.+\left[(1-\delta)\boldsymbol{I} - \bar{R}^k\boldsymbol{L}\right]\frac{\psi}{\bar{K}}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{D}^{-1}\right] + (\lambda\bar{K}^u - (1-\lambda)\bar{K}^c)\frac{\psi}{\bar{K}}(\boldsymbol{I}-\boldsymbol{L})\\
&+\left(-\boldsymbol{\Theta}(\lambda) + \frac{1-\lambda}{\bar{\eta}}\left(\boldsymbol{\Theta}_B^c + \boldsymbol{\Theta}_F^c\right) - \lambda\boldsymbol{B}\right)\frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I})\times\left[\frac{(1+\varphi)(1-\alpha)\bar{Y}}{\left(1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}\right)\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\right.\\
&\left.\left.-\frac{1+\varphi-\alpha}{1+\varphi-\alpha+\alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}}\boldsymbol{I}\right]\right\}\mathrm{d}\boldsymbol{\mathcal{I}} \qquad \text{(A.35)}\\
&+\left\{\boldsymbol{\Theta}(\lambda)\frac{\alpha\gamma^{\mathrm{HH}}\bar{r}^k}{\widetilde{\phi}\,\bar{C}} - \left(-\boldsymbol{\Theta}(\lambda) + \frac{1-\lambda}{\bar{\eta}}\left(\boldsymbol{\Theta}_B^c + \boldsymbol{\Theta}_F^c\right) - \lambda\boldsymbol{B}\right)\frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I})\frac{1+\varphi-\alpha}{\widetilde{\phi}}\right\}\mathrm{d}\boldsymbol{D}
\end{aligned}$$

(A.36)

The expression above gives the loadings of $\mathrm{d}\boldsymbol{\mathcal{I}}$ on $\mathrm{d}\boldsymbol{A}$, $\mathrm{d}\boldsymbol{\mathcal{I}}$ and $\mathrm{d}\boldsymbol{D}$. Plugging (A.23), (A.27) and (A.29) in (A.33) gives those of $\mathrm{d}\boldsymbol{D}$. Stacking the two expressions and rearranging gives (A.20). □

A.8 Corollary A.1: Relaxing the Assumptions

We now extend Corollary A.1 to the case where we relax the three simplifying assumptions in the main text. As in the main text, we consider a pure belief shock: unconstrained intermediaries hold arbitrary beliefs $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}$ about the path of future TFP, while the realized path of productivity remains at its steady-state value, $\mathrm{d}\boldsymbol{A} = \boldsymbol{0}$. Households continue to form expectations under FIRE.

We maintain the assumption that unconstrained intermediaries use the model-consistent FIRE mapping from TFP into aggregate outcomes to form their expectations about endogenous variables. From (A.20), define the closed-economy FIRE investment Jacobian

$$\boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda) \equiv [\boldsymbol{I} - \boldsymbol{\mathcal{G}}(\lambda)]^{-1} \boldsymbol{\mathcal{A}}(\lambda). \tag{A.37}$$

Where CE stands for closed-economy. Unconstrained intermediaries then perceive aggregate investment to satisfy

$$\begin{pmatrix} \widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{\mathcal{I}} \\ \widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{D} \end{pmatrix} = \boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}. \tag{A.38}$$

Given their beliefs $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}, \widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{\mathcal{I}}$ unconstrained intermediaries follow expectations about future rental rates and risk-free rates. As in the main text, we assume intermediaries observe the current price of capital. We also assume that the one-period risk-free rate R_{t+1} is observed when capital holdings at date t are chosen.

Proposition A.2. *Consider a shock to unconstrained intermediaries' beliefs about future TFP, $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}$, holding fixed the realized path of productivity. The general-equilibrium investment response in a closed economy is*

$$\begin{aligned}\begin{pmatrix} \mathrm{d}\boldsymbol{\mathcal{I}} \\ \mathrm{d}\boldsymbol{D} \end{pmatrix} =& \boldsymbol{\mathcal{M}}^{\mathrm{CE}}(\lambda)\lambda\Bigg\{\widetilde{\boldsymbol{X}}\Bigg[\frac{(1+\varphi)\bar{r}^k}{\widetilde{\phi}\bar{A}}\boldsymbol{I} + \Bigg(\frac{\alpha\gamma^{\mathrm{HH}}\bar{r}^k}{\widetilde{\phi}\bar{C}}\boldsymbol{J} - \frac{\alpha\left(\varphi+\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}\right)\bar{r}^k}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S} \\ &+ (1-\delta)\frac{\psi}{\bar{K}}\boldsymbol{D}^{-1}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{S}\Bigg)\boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\Bigg] \\ &+ \widetilde{\boldsymbol{Z}}\frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I})\Bigg[\frac{(1+\varphi)\bar{Y}}{\widetilde{\phi}\bar{A}}\boldsymbol{I} + \Bigg(\frac{(1+\varphi)(1-\alpha)\bar{Y}}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S} - \frac{1+\varphi-\alpha}{\widetilde{\phi}}\boldsymbol{J}\Bigg)\boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\Bigg]\Bigg\}\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A},\end{aligned} \tag{A.39}$$

where

$$\begin{aligned}\boldsymbol{\mathcal{M}}^{\mathrm{CE}}(\lambda) \equiv \Bigg[&\boldsymbol{I} - \boldsymbol{\mathcal{G}}(\lambda) + \lambda\widetilde{\boldsymbol{X}}\Bigg(\frac{\alpha\gamma^{\mathrm{HH}}\bar{r}^k}{\widetilde{\phi}\bar{C}}\boldsymbol{J} - \frac{\alpha\left(\varphi+\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}\right)\bar{r}^k}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S} + (1-\delta)\frac{\psi}{\bar{K}}\boldsymbol{D}^{-1}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{S}\Bigg) \\ &+ \lambda\widetilde{\boldsymbol{Z}}\frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I})\Bigg(\frac{(1+\varphi)(1-\alpha)\bar{Y}}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S} - \frac{1+\varphi-\alpha}{\widetilde{\phi}}\boldsymbol{J}\Bigg)\Bigg]^{-1}.\end{aligned} \tag{A.40}$$

with $\boldsymbol{S}$, $\boldsymbol{J}$, $\boldsymbol{\Gamma}$ *and* $\boldsymbol{\Pi}$ *as in* (A.20),

$$\widetilde{\boldsymbol{X}} \equiv \begin{pmatrix} \boldsymbol{\Theta}^u \\ \widetilde{\boldsymbol{\Gamma}}\boldsymbol{F} \end{pmatrix}, \qquad \widetilde{\boldsymbol{Z}} \equiv \begin{pmatrix} -\boldsymbol{B} \\ \widetilde{\boldsymbol{\Pi}} \end{pmatrix}, \qquad \widetilde{\boldsymbol{\Gamma}} \equiv \boldsymbol{\Gamma} - \bar{\varsigma}\bar{K}^u\boldsymbol{N}, \qquad \widetilde{\boldsymbol{\Pi}} \equiv \boldsymbol{\Pi} - \bar{\varsigma}\left(\bar{N}^u - \bar{D}^u\right)\boldsymbol{N}\boldsymbol{F},$$

and $\widetilde{\phi} \equiv 1 + \varphi - \alpha + \alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}$.

Proof. Using (A.23) and (A.38), intermediaries' perceived path for the price of capital is

$$\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{q} = \frac{\psi}{\bar{K}}\boldsymbol{D}^{-1}(\boldsymbol{I} - \boldsymbol{L})\boldsymbol{S}\,\boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}. \tag{A.41}$$

Using (A.27), and denoting $\widetilde{\phi} \equiv 1 + \varphi - \alpha + \alpha\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}$, the perceived path for the rental rate is

$$\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{r}^k = \left[\frac{(1+\varphi)\bar{r}^k}{\widetilde{\phi}\bar{A}}\boldsymbol{I} + \left(\frac{\alpha\gamma^{\mathrm{HH}}\bar{r}^k}{\widetilde{\phi}\bar{C}}\boldsymbol{J} - \frac{\alpha\left(\varphi + \gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}\right)\bar{r}^k}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S}\right)\boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\right]\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}. \tag{A.42}$$

Similarly, using (A.26), their perceived path for household consumption is

$$\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{C} = \left[\frac{(1+\varphi)\bar{Y}}{\widetilde{\phi}\bar{A}}\boldsymbol{I} + \left(\frac{(1+\varphi)(1-\alpha)\bar{Y}}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S} - \frac{1+\varphi-\alpha}{\widetilde{\phi}}\boldsymbol{J}\right)\boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\right]\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}. \tag{A.43}$$

It follows from the household Euler equation that the perceived path for future risk-free rates is

$$\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{R} = \frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F} - \boldsymbol{I})\left[\frac{(1+\varphi)\bar{Y}}{\widetilde{\phi}\bar{A}}\boldsymbol{I} + \left(\frac{(1+\varphi)(1-\alpha)\bar{Y}}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S} - \frac{1+\varphi-\alpha}{\widetilde{\phi}}\boldsymbol{J}\right)\boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\right]\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} \tag{A.44}$$

We next characterize actual equilibrium outcomes. Since the realized TFP path is unchanged, $\mathrm{d}\boldsymbol{A} = \boldsymbol{0}$, writing $\mathrm{d}\boldsymbol{W} \equiv \left(\mathrm{d}\boldsymbol{\mathcal{I}},\ \mathrm{d}\boldsymbol{D}\right)'$, equations (A.23), (A.27), (A.26), and (A.29) imply that the path of the price of capital is

$$\mathrm{d}\boldsymbol{q} = \frac{\psi}{\bar{K}}\boldsymbol{D}^{-1}(\boldsymbol{I} - \boldsymbol{L})\boldsymbol{S}\,\mathrm{d}\boldsymbol{W}, \tag{A.45}$$

and the rental rate of capital follows

$$\mathrm{d}\boldsymbol{r}^k = \left[\frac{\alpha\gamma^{\mathrm{HH}}\bar{r}^k}{\widetilde{\phi}\bar{C}}\boldsymbol{J} - \frac{\alpha\left(\varphi + \gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}\right)\bar{r}^k}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S}\right]\mathrm{d}\boldsymbol{W}, \tag{A.46}$$

and consumption

$$\mathrm{d}\boldsymbol{C} = \left[\frac{(1+\varphi)(1-\alpha)\bar{Y}}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S} - \frac{1+\varphi-\alpha}{\widetilde{\phi}}\boldsymbol{J}\right]\mathrm{d}\boldsymbol{W}, \tag{A.47}$$

and finally the risk-free rate

$$\mathrm{d}\boldsymbol{R} = \frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I})\left[\frac{(1+\varphi)(1-\alpha)\bar{Y}}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S} - \frac{1+\varphi-\alpha}{\widetilde{\phi}}\boldsymbol{J}\right]\mathrm{d}\boldsymbol{W}. \tag{A.48}$$

Because unconstrained intermediaries observe the current capital price and the one-period risk-free rate, but form subjective beliefs about the future rental rate, future resale price, and the future risk-free rates entering their continuation risk tolerance, their investment satisfies

$$\mathrm{d}\boldsymbol{\mathcal{I}}^u = \boldsymbol{\Theta}^u\,\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{\mu} + \bar{K}^u\boldsymbol{D}\,\mathrm{d}\boldsymbol{q} - \boldsymbol{B}\,\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{R}. \tag{A.49}$$

Because households have FIRE beliefs, constrained intermediaries respond to actual returns and actual risk-free rates:

$$\mathrm{d}\boldsymbol{\mathcal{I}}^c = (\boldsymbol{\Theta}^c_B + \boldsymbol{\Theta}^c_F)\left[\mathrm{d}\boldsymbol{r}^k + (1-\delta)\mathrm{d}\boldsymbol{q} - \bar{R}^k\boldsymbol{L}\,\mathrm{d}\boldsymbol{q} + \left(\frac{1}{\bar{\eta}}-1\right)\mathrm{d}\boldsymbol{R}\right] - \bar{K}^c\boldsymbol{D}\,\mathrm{d}\boldsymbol{q}. \tag{A.50}$$

Splitting $\boldsymbol{\Gamma}$ and $\boldsymbol{\Pi}$ accordingly, (A.33) becomes

$$\begin{aligned}\mathrm{d}\boldsymbol{D} = \lambda\Big[&\widetilde{\boldsymbol{\Gamma}}\big(\boldsymbol{F}\,\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{\mu} + \bar{\mu}\,\mathrm{d}\boldsymbol{q}\big) + \bar{\varsigma}\bar{K}^u\boldsymbol{N}\big(\boldsymbol{F}\,\mathrm{d}\boldsymbol{\mu} + \bar{\mu}\,\mathrm{d}\boldsymbol{q}\big) + \widetilde{\boldsymbol{\Pi}}\,\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{R} + \bar{\varsigma}\big(\bar{N}^u - \bar{D}^u\big)\boldsymbol{N}\boldsymbol{F}\,\mathrm{d}\boldsymbol{R}\Big]\\ &+ (1-\lambda)\,\boldsymbol{\Gamma}^c\Big(\mathrm{d}\boldsymbol{\mu} + \tfrac{1}{\bar{\eta}}\mathrm{d}\boldsymbol{R}\Big),\end{aligned} \tag{A.51}$$

where the constrained block is at realized returns throughout, since constrained intermediaries' leverage is set by their depositors, who hold FIRE beliefs.

Substituting (A.41)–(A.51) in the definitions of aggregate investment and aggregate dividends we obtain

$$\begin{aligned}\begin{pmatrix}\mathrm{d}\boldsymbol{\mathcal{I}}\\ \mathrm{d}\boldsymbol{D}\end{pmatrix} =\lambda\Bigg\{&\widetilde{\boldsymbol{X}}\left[\frac{(1+\varphi)\bar{r}^k}{\widetilde{\phi}\bar{A}}\boldsymbol{I} + \left(\frac{\alpha\gamma^{\mathrm{HH}}\bar{r}^k}{\widetilde{\phi}\bar{C}}\boldsymbol{J} - \frac{\alpha\left(\varphi+\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}\right)\bar{r}^k}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S} + (1-\delta)\frac{\psi}{\bar{K}}\boldsymbol{D}^{-1}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{S}\right)\boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\right]\\ &+\widetilde{\boldsymbol{Z}}\frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I})\left[\frac{(1+\varphi)\bar{Y}}{\widetilde{\phi}\bar{A}}\boldsymbol{I} + \left(\frac{(1+\varphi)(1-\alpha)\bar{Y}}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S} - \frac{1+\varphi-\alpha}{\widetilde{\phi}}\boldsymbol{J}\right)\boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\right]\Bigg\}\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}\end{aligned}$$

$$+\left\{\boldsymbol{\mathcal{G}}(\lambda)-\lambda\widetilde{\boldsymbol{X}}\left(\frac{\alpha\gamma^{\mathrm{HH}}\bar{r}^{k}}{\widetilde{\phi}\bar{C}}\boldsymbol{J}-\frac{\alpha\left(\varphi+\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}\right)\bar{r}^{k}}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S}+(1-\delta)\frac{\psi}{\bar{K}}\boldsymbol{D}^{-1}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{S}\right)\right.$$
$$\left.-\lambda\widetilde{\boldsymbol{Z}}\frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I})\left(\frac{(1+\varphi)(1-\alpha)\bar{Y}}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S}-\frac{1+\varphi-\alpha}{\widetilde{\phi}}\boldsymbol{J}\right)\right\}\begin{pmatrix}\mathrm{d}\boldsymbol{\mathcal{I}}\\ \mathrm{d}\boldsymbol{D}\end{pmatrix}. \tag{A.52}$$

Rearranging gives (A.39). □

A.9 Proposition 2: Relaxing the Assumptions

We now extend Proposition 2 to the case where we relax the three simplifying assumptions in the main text. We maintain the assumptions on beliefs and information introduced in the previous subsection. In particular, households continue to form expectations under FIRE, while unconstrained intermediaries use the counterfactual closed-economy FIRE equilibrium associated with their subjective TFP path to form expectations about endogenous variables. Moreover, unconstrained intermediaries observe the current capital price and the one-period risk-free rate when choosing their capital holdings.

Corollary A.2. *Suppose that the operators defining $\boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)$ and $\boldsymbol{\mathcal{M}}^{\mathrm{CE}}(\lambda)$ are boundedly invertible. The general-equilibrium investment response to a realized TFP path $\mathrm{d}\boldsymbol{A}$, given an arbitrary perceived TFP path $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}$, is*

$$\begin{pmatrix}\mathrm{d}\boldsymbol{\mathcal{I}}\\ \mathrm{d}\boldsymbol{D}\end{pmatrix}=\boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\mathrm{d}\boldsymbol{A}+\widetilde{\boldsymbol{\mathcal{J}}}^{\mathrm{CE}}(\lambda)\left(\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}-\mathrm{d}\boldsymbol{A}\right), \tag{A.53}$$

where $\boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)$ and $\boldsymbol{\mathcal{M}}^{\mathrm{CE}}(\lambda)$ are defined in (A.37) and (A.40), respectively, and

$$\begin{aligned}\widetilde{\boldsymbol{\mathcal{J}}}^{\mathrm{CE}}(\lambda)\equiv\boldsymbol{\mathcal{M}}^{\mathrm{CE}}(\lambda)\lambda\Bigg\{&\widetilde{\boldsymbol{X}}\left[\frac{(1+\varphi)\bar{r}^{k}}{\widetilde{\phi}\bar{A}}\boldsymbol{I}+\left(\frac{\alpha\gamma^{\mathrm{HH}}\bar{r}^{k}}{\widetilde{\phi}\bar{C}}\boldsymbol{J}-\frac{\alpha\left(\varphi+\gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}\right)\bar{r}^{k}}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S}\right.\right.\\&\left.\left.+(1-\delta)\frac{\psi}{\bar{K}}\boldsymbol{D}^{-1}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{S}\right)\boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\right]\\&+\widetilde{\boldsymbol{Z}}\frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I})\left[\frac{(1+\varphi)\bar{Y}}{\widetilde{\phi}\bar{A}}\boldsymbol{I}+\left(\frac{(1+\varphi)(1-\alpha)\bar{Y}}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S}-\frac{1+\varphi-\alpha}{\widetilde{\phi}}\boldsymbol{J}\right)\boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\right]\Bigg\}.\end{aligned} \tag{A.54}$$

When unconstrained intermediaries' expectation formation process satisfies $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}=\chi\mathrm{d}\boldsymbol{A}$, where χ is a

scalar, (A.53) *becomes*

$$\begin{pmatrix} \mathrm{d}\boldsymbol{\mathcal{I}} \\ \mathrm{d}\boldsymbol{D} \end{pmatrix} = \boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\mathrm{d}\boldsymbol{A} + (\chi - 1)\widetilde{\boldsymbol{\mathcal{J}}}^{\mathrm{CE}}(\lambda)\mathrm{d}\boldsymbol{A}. \tag{A.55}$$

Proof. Let households' expectations satisfy $\mathbb{E}\,\mathrm{d}\boldsymbol{A} = \mathrm{d}\boldsymbol{A}$ and $\mathbb{E}\,\mathrm{d}\boldsymbol{\mathcal{I}} = \mathrm{d}\boldsymbol{\mathcal{I}}$, while unconstrained intermediaries perceive aggregate investment and dividends to satisfy

$$\begin{pmatrix} \widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{\mathcal{I}} \\ \widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{D} \end{pmatrix} = \boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A}.$$

Let $\mathrm{d}\boldsymbol{W}^{\mathrm{FIRE}}$ denote the response when unconstrained intermediaries also hold FIRE beliefs. From (A.20),

$$\mathrm{d}\boldsymbol{W}^{\mathrm{FIRE}} = \boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\mathrm{d}\boldsymbol{A}. \tag{A.56}$$

Define

$$\boldsymbol{Z} \equiv \mathrm{d}\boldsymbol{W} - \mathrm{d}\boldsymbol{W}^{\mathrm{FIRE}}, \qquad \mathrm{d}\boldsymbol{W} \equiv \left(\mathrm{d}\boldsymbol{\mathcal{I}},\, \mathrm{d}\boldsymbol{D}\right)'.$$

Using (A.23), (A.27), (A.26), (A.29) and (A.51), together with the perceived paths derived in (A.41)–(A.44), subtracting the FIRE equilibrium from the equilibrium under arbitrary beliefs gives

$$\begin{aligned}
\boldsymbol{Z} = \lambda \Bigg\{ & \widetilde{\boldsymbol{X}} \left[\frac{(1+\varphi)\bar{r}^k}{\widetilde{\phi}\bar{A}}\boldsymbol{I} + \left(\frac{\alpha\gamma^{\mathrm{HH}}\bar{r}^k}{\widetilde{\phi}\bar{C}}\boldsymbol{J} - \frac{\alpha\left(\varphi + \gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}\right)\bar{r}^k}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S} + (1-\delta)\frac{\psi}{\bar{K}}\boldsymbol{D}^{-1}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{S} \right) \boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda) \right] \\
& + \widetilde{\boldsymbol{Z}}\frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I}) \left[\frac{(1+\varphi)\bar{Y}}{\widetilde{\phi}\bar{A}}\boldsymbol{I} + \left(\frac{(1+\varphi)(1-\alpha)\bar{Y}}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S} - \frac{1+\varphi-\alpha}{\widetilde{\phi}}\boldsymbol{J} \right) \boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda) \right] \Bigg\} \left(\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} - \mathrm{d}\boldsymbol{A} \right) \\
& + \Bigg\{ \boldsymbol{\mathcal{G}}(\lambda) - \lambda\widetilde{\boldsymbol{X}} \left(\frac{\alpha\gamma^{\mathrm{HH}}\bar{r}^k}{\widetilde{\phi}\bar{C}}\boldsymbol{J} - \frac{\alpha\left(\varphi + \gamma^{\mathrm{HH}}\frac{\bar{Y}}{\bar{C}}\right)\bar{r}^k}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S} + (1-\delta)\frac{\psi}{\bar{K}}\boldsymbol{D}^{-1}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{S} \right) \\
& - \lambda\widetilde{\boldsymbol{Z}}\frac{\gamma^{\mathrm{HH}}\bar{R}}{\bar{C}}\boldsymbol{L}(\boldsymbol{F}-\boldsymbol{I}) \left(\frac{(1+\varphi)(1-\alpha)\bar{Y}}{\widetilde{\phi}\bar{K}}\boldsymbol{L}\boldsymbol{D}^{-1}\boldsymbol{S} - \frac{1+\varphi-\alpha}{\widetilde{\phi}}\boldsymbol{J} \right) \Bigg\} \boldsymbol{Z}.
\end{aligned} \tag{A.57}$$

Rearranging and using (A.54) therefore gives

$$\boldsymbol{Z} = \widetilde{\boldsymbol{\mathcal{J}}}^{\mathrm{CE}}(\lambda) \left(\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} - \mathrm{d}\boldsymbol{A} \right). \tag{A.58}$$

Finally, using the definition of $\boldsymbol{Z}$ and $\mathrm{d}\boldsymbol{W}^{\mathrm{FIRE}} = \boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\mathrm{d}\boldsymbol{A}$, we obtain

$$\begin{pmatrix} \mathrm{d}\boldsymbol{\mathcal{I}} \\ \mathrm{d}\boldsymbol{D} \end{pmatrix} = \boldsymbol{\mathcal{J}}^{\mathrm{CE}}(\lambda)\mathrm{d}\boldsymbol{A} + \widetilde{\boldsymbol{\mathcal{J}}}^{\mathrm{CE}}(\lambda)\left(\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} - \mathrm{d}\boldsymbol{A}\right), \tag{A.59}$$

which proves (A.53). Imposing $\widetilde{\mathbb{E}}\mathrm{d}\boldsymbol{A} = \chi\mathrm{d}\boldsymbol{A}$ immediately gives (A.55).

□

B Appendix to Section 3

In this Section we present additional figures and tables, as well as robustness exercises, related to our empirical exercises from Section 3.

B.1 Additional Tables

Summary statistics. Table B.1 presents summary statistics for the AnaCredit dataset used in our analysis (panels (a) and (b)) as well as for the merged AnaCredit-IBSI and AnaCredit-EBA datasets (panel (c)).

Table B.1 – Summary Statistics

	Mean	SD	P10	P25	P50	P75	P90	N
Panel (a): Reported default probabilities								
PD (%)	3.57	8.19	0.18	0.40	1.07	2.96	7.85	34,383,338
Quarterly ΔPD (pp)	0.13	2.39	-0.48	0.00	0.00	0.00	0.58	32,024,690
PD update	0.402	0.490	0	0	0	1	1	32,024,690
Panel (b): Credit relationships								
Total exposure (EUR million)	1.54	22.86	0.04	0.07	0.19	0.53	1.65	33,254,970
Interest rate (%)	3.17	2.24	0.53	1.16	2.31	4.07	5.87	31,547,198
Remaining maturity (years)	3.76	3.58	0.33	1.27	2.92	4.73	8.57	31,172,418
Panel (c): Bank characteristics								
Total assets (EUR billion)	66.00	113.14	3.74	8.69	24.82	58.95	187.45	3,534
Total loans (EUR billion)	26.08	75.05	0.39	1.98	5.90	14.73	45.62	3,534
Loans / assets (%)	28.50	21.46	6.25	14.76	22.68	35.67	60.64	3,534
CET1 / risk-weighted assets (%)	15.22	3.58	12.23	13.03	14.63	16.23	18.88	748
Regulatory leverage ratio (%)	5.65	3.69	4.05	4.47	5.13	5.96	7.08	748
Pillar 1 capital slack (pp)	10.12	3.37	7.48	8.22	9.34	11.02	13.67	748

Notes: panels (a) and (b) report summary statistics for our AnaCredit subsample, where we aggregate observations at the bank-firm-quarter level. PD update is a binary variable that takes the value of one if the reported PD is different from the one reported in the previous quarter. Total exposure includes outstanding nominal and off-balance-sheet amounts. Interest rates and maturity are relationship-level averages, weighted by loan size. Panel (c) uses the merged AnaCredit-IBSI and AnaCredit-EBA samples. The unit of observation is a bank–quarter observations: the first three rows use IBSI and the remaining rows use EBA. Pillar 1 slack is the minimum distance above the CET1, Tier 1 and total capital minima of 4.5%, 6% and 8% of risk-weighted assets, respectively. SD denotes standard deviation and pp denotes percentage points.

Cross-sectional variation in reported PDs. Table B.2 reports the shares of variation in reported default probabilities and their quarterly changes accounted for by differences across

lenders within firm–quarters and differences between firm–quarters. Let y_{bft} denote either reported PDs or their quarterly changes, $\bar{y}_{ft}$ the average across the n_{ft} lenders to firm f in quarter t, and $\bar{y}$ the average across all $N = \sum_{f,t} n_{ft}$ observations. We define

$$V_{\text{between}} = \frac{1}{N}\sum_{f,t} n_{ft}(\bar{y}_{ft} - \bar{y})^2, \qquad V_{\text{within}} = \frac{1}{N}\sum_{f,t}\sum_{b}(y_{bft} - \bar{y}_{ft})^2.$$

The table reports $100 \times \frac{V_{\text{between}}}{V_{\text{between}}+V_{\text{within}}}$ and $100 \times \frac{V_{\text{within}}}{V_{\text{between}}+V_{\text{within}}}$.

We report the bank–quarter share as $100 \times \frac{V_{\text{within}}-V_u}{V_{\text{between}}+V_{\text{within}}}$, where $V_u = \frac{1}{N}\sum_{f,t}\sum_{b}\hat{u}^2_{bft}$ is the mean squared residual from the following regression, estimated on the same sample:

$$y_{bft} = \gamma_{ft} + \alpha_{bt} + u_{bft},$$

where γ_{ft} and α_{bt} are firm-quarter and bank-quarter fixed effects.

Table B.2 – Dispersion of Reported Default Probabilities

	PD	Quarterly ΔPD
Standard deviation (pp)	8.19	2.39
Share of total variance (%)		
Between firm-quarters	82.55	48.41
Within firm-quarter	17.45	51.59
Of which: bank-quarter component	0.97	3.63

Notes: PD is the reported one-year probability of default. Quarterly ΔPD is its change relative to the preceding quarter for the same bank–firm relationship. For each firm-quarter, we calculate the mean and variance of PDs and ΔPD across lenders. The between component is the variance of these firm-quarter means. The within component is the average of the firm-quarter variances. The bank-quarter component is the difference between the within-firm-quarter variance and the residual variance after jointly accounting for firm-quarter and bank-quarter fixed effects. All components are expressed as shares of the respective variable's total variance.

Table B.3 decomposes the variation in quarterly PD revisions within firm–quarters into an extensive component, capturing variation in whether lenders update their PDs of the same firm, and an intensive component, capturing differences in how much they revise their PDs conditional on updating. Let p_{ft} denote the share of lenders updating firm f's PD in quarter t, and let m_{ft} and σ^2_{ft} denote the mean and variance of revisions among updating lenders. Since

non-updating lenders have zero revisions, within-firm-quarter variance satisfies

$$\text{Var}(\Delta PD_{bft} \mid f, t) = \underbrace{p_{ft}(1 - p_{ft})m_{ft}^2}_{\text{Extensive}} + \underbrace{p_{ft}\sigma_{ft}^2}_{\text{Intensive}} .$$

The first term measures the variance that would remain if each updating lender made the same revision, equal to the average among updating lenders in that firm-quarter; the second captures dispersion around that average among updating lenders. We aggregate both terms across firm-quarters, weighting by the number of lenders, and report each as a share of aggregate within-firm-quarter variance.

Table B.3 – Frequency of PD Updating and Intensive vs Extensive Margin of Variation

	%
Panel A: Frequency and coordination of updating	
Update frequency	40.27
Share of total variance	
Between firm-quarters	47.62
Within firm-quarter	52.38
Of which: bank-quarter component	14.34
Panel B: Extensive and intensive margins	
Share of within-firm-quarter variance of ΔPD	
Extensive margin	57.12
Intensive margin	42.88

Notes: Panel A decomposes the variance of an indicator equal to one when a lender updates a firm's PD relative to the preceding quarter. The between-firm-quarter component is the variance of average updating frequency across firm-quarters. The within-firm-quarter component is the average variance of updating decisions across lenders within the same firm-quarter. Both components are expressed as shares of total indicator variance. The bank-quarter component is the difference between the within-firm-quarter variance and the residual variance after jointly accounting for firm-quarter and bank-quarter fixed effects, divided by total indicator variance. Panel B decomposes the within-firm-quarter variance of quarterly PD revisions into extensive and intensive components. Within each firm-quarter, let p denote the share of lenders updating and let m and σ^2 denote the mean and variance of revisions among updating lenders. We define the extensive component as $p(1-p)m^2$, capturing variation between updating and non-updating lenders, assigning all updating lenders their average revision. We define the intensive component as $p\sigma^2$, capturing dispersion in revisions among updating lenders. Components are aggregated across firm-quarters and reported as shares of aggregate within-firm-quarter variance.

Borrower reallocation across lenders. Table B.4 compares lending responses estimated at the bank–firm and firm levels. We consider two measures of credit: outstanding loans and total

exposure, defined as outstanding loans plus undrawn commitments. We measure cumulative growth asOur outcome variable is cumulative DHS growth, defined as:

$$\Delta\text{Loans}_{bft}(h) = 2 \times \frac{L_{bf,t+h} - L_{bf,t-1}}{L_{bf,t+h} + L_{bf,t-1}}, \tag{B.1}$$

where L_{bft} denote the relevant loan measure for bank b and firm f in quarter t.

Panel A reports estimates from the following bank–firm-level local projection at quarterly horizons $h \in 0, 1, 2, 3, 4$:

$$\Delta\text{Loans}_{bft}(h) = \xi_{bfh} + \gamma_{fth} + \beta_h^B \widehat{\alpha}_{bt} + \Xi_h^{B\prime} \boldsymbol{X}_{bf,t-1} + u_{bfth}, \tag{B.2}$$

where ξ_{bfh} denotes bank–firm fixed effects, γ_{fth} denotes firm–quarter fixed effects (as in Khwaja and Mian (2008)), and the vector $\boldsymbol{X}_{bf,t-1}$ contains four lags of one-quarter DHS growth in the corresponding lending measure.

For Panel B, we aggregate lending across all banks, $L_{ft} = \sum_b L_{bft}$, and construct the firm's exposure to bank shocks as

$$Z_{ft} = \sum_{b \in \mathcal{B}_{f,t-1}} w_{bf,t-1} \widehat{\alpha}_{bt}, \qquad w_{bf,t-1} = \frac{L_{bf,t-1}}{\sum_{k \in \mathcal{B}_{f,t-1}} L_{kf,t-1}}, \tag{B.3}$$

where $\mathcal{B}_{f,t-1}$ is the set of lenders firm f borrowed with in quarter $t-1$. We construct the weights separately for outstanding lending and total exposure, using the same measure as the dependent variable.

We then estimate

$$\Delta\text{Loans}_{ft}(h) = \gamma_{fh} + \delta_{c(f),t,h} + \lambda_{s(f),t,h} + \beta_h^F Z_{ft} + \Xi_h^{F\prime} \boldsymbol{X}_{f,t-1} + u_{fth}, \tag{B.4}$$

where $\Delta\text{Loans}_{ft}(h)$ is cumulative DHS growth in the firm's total borrowing, defined analogously to (B.1), $\delta_{c(f),t,h}$ is a country–quarter fixed effect, and $\lambda_{s(f),t,h}$ denotes a two-digit industry–quarter fixed effect. The controls contain four lags of firm shock exposure and four lags of one-quarter DHS growth in total borrowing.

Table B.4 – Reallocation across lenders

Horizon (quarters)	0	1	2	3	4
Panel A. Bank–firm level					
Outstanding lending	0.085***	0.108***	0.205***	0.144***	0.157***
	(0.008)	(0.009)	(0.010)	(0.011)	(0.011)
Total exposure	0.111***	0.120***	0.177***	0.121***	0.169***
	(0.006)	(0.008)	(0.010)	(0.010)	(0.011)
Observations: outstanding	20,898,190	19,019,163	17,422,476	16,038,158	14,775,604
Observations: exposure	15,062,186	13,718,256	12,546,949	11,533,947	10,614,439
Bank–firm fixed effects	Yes	Yes	Yes	Yes	Yes
Firm–quarter fixed effects	Yes	Yes	Yes	Yes	Yes
Panel B. Firm level					
Outstanding lending	0.089***	0.145***	0.180***	0.175***	0.131***
	(0.015)	(0.018)	(0.021)	(0.023)	(0.025)
Total exposure	0.092***	0.117***	0.143***	0.095***	0.140***
	(0.014)	(0.018)	(0.021)	(0.024)	(0.026)
Observations: outstanding	5,585,206	4,889,647	4,317,427	3,862,309	3,465,143
Observations: exposure	3,197,748	2,807,035	2,469,476	2,202,068	1,967,813
Firm fixed effects	Yes	Yes	Yes	Yes	Yes
Country–quarter fixed effects	Yes	Yes	Yes	Yes	Yes
Industry–quarter fixed effects	Yes	Yes	Yes	Yes	Yes

Notes: panels A reports bank–firm level estimates of β_h^B from (B.2), while panel B reports firm–level estimates of β_h^F from (B.4). The dependent variable is cumulative DHS growth between quarters $t-1$ and $t+h$ in outstanding loans or total exposure, defined as outstanding loans plus undrawn commitments. In Panel B, firm exposure to bank shocks is a weighted average of lenders' shocks, with weights given by each lender's share of the firm's credit in quarter $t-1$. Both specifications include four lags of one-quarter DHS growth in the corresponding credit measure; Panel B additionally includes four lags of firm exposure to bank shocks. Standard errors are reported in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Response of firm-level employment. In Table B.5 we estimate the dynamic response of firm employment to the average expectation shocks of its lenders. Since employment is reported annually, we aggregate the quarterly bank shocks and estimate local projections at annual frequency. Let $\widehat{\alpha}_{btq}$ denote bank b's shock in quarter q of calendar year t. We construct its annual

shock as measure firm f's exposure as a weighted average of its lenders' annual shocks:

$$Z_{ft} = \sum_{b \in \mathcal{B}_{f,t-1}} w_{bf,t-1} \sum_{q=1}^{4} \widehat{\alpha}_{btq}., \qquad w_{bf,t-1} = \frac{L_{bf,t-1}}{\sum_{k \in \mathcal{B}_{f,t-1}} L_{kf,t-1}}, \tag{B.5}$$

where $L_{bf,t-1}$ is the average outstanding loans from bank b to firm f in year $t-1$, and $\mathcal{B}_{f,t-1}$ is the set of lenders firm f borrowed with in that year.

For each yearly horizon $h \in \{0, 1, 2\}$, we then estimate:

$$\Delta \text{Emp}_{ft+h} = \gamma_{fh} + \delta_{c(f),t,h} + \lambda_{s(f),t,h} + \beta_h Z_{ft} + \rho_h Z_{f,t-1} + \phi_h \Delta \text{Emp}_{ft-1} + u_{fth}, \tag{B.6}$$

where γ_{fh} is a firm fixed effect, $\delta_{c(f),t,h}$ denotes a country–year fixed effect and $\lambda_{s(f),t,h}$ a 2-digit industry–year fixed effect. ΔEmp_{ft+h} is cumulative DHS employment growth between $t-1$ and $t+h$, defined as $2 \times \frac{\text{Emp}_{ft+h} - \text{Emp}_{ft-1}}{\text{Emp}_{ft+h} + \text{Emp}_{ft-1}}$.

Table B.5 – Employment Response to $\widehat{\alpha}_{bt}$

Horizon (years)	0	1	2
Employment: cumulative DHS growth	0.037***	0.108***	0.082***
	(0.012)	(0.038)	(0.027)
Observations	1,564,418	1,075,271	694,454
Firm fixed effects	Yes	Yes	Yes
Country–year fixed effects	Yes	Yes	Yes
Industry–year fixed effects	Yes	Yes	Yes
Lagged controls	Yes	Yes	Yes

Notes: the table reports estimates of β_h from the annual local projections in (B.6), for $h \in \{0, 1, 2\}$. The firm-level shock is defined according to (B.5), averaging lenders' annual shocks using loan shares from the preceding year. All specifications include firm, country–year, and two-digit industry–year fixed effects. Cumulative DHS growth is $2 \times \left(\text{Emp}_{f,t+h} - \text{Emp}_{f,t-1}\right) / \left(\text{Emp}_{f,t+h} + \text{Emp}_{f,t-1}\right)$. Standard errors in parenthesis. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

B.2 Additional Figures

Figure B.1 plots aggregate realized default rates among non-financial corporate borrowers of German and French commercial banks over 2005–2013, using semiannual data from Dietsch et al. (2016). Default is defined according to Basel II in Germany and the local judicial definition

in France.

Figure B.2 shows the number of borrowers and lenders available for comparing banks' forecast revisions for the same firms. For each bank-quarter, panel (a) counts distinct borrowers for which both the bank and another lender report usable PD revisions. Panel (b) reports the number of distinct other banks reporting a usable PD update for at least one of these borrowers in the same quarter.

Figure B.3 compares estimates of the $\widehat{\alpha}_{bt}$ shock estimated using (21) on different borrower samples. Panels (a) and (b) compare the estimated bank effects for manufacturing borrowers with those for trade and transport borrowers, respectively. Panel (c) compares borrowers with baseline PDs at or below the median with those above it. Panel (d) compares estimates based on bank–firm relationships with and without collateral.

Figure B.4 compares estimates obtained by re-estimating (21), replacing bank–time fixed effects with bank–time–sector fixed effects in panel (a) and bank–time–region fixed effects in panel (b). For each bank-quarter, panel (a) compares the bank's estimated effect in each two-digit sector with the equally weighted average of its effects in the other sectors. Panel (b) repeats this comparison across NUTS 2 regions.

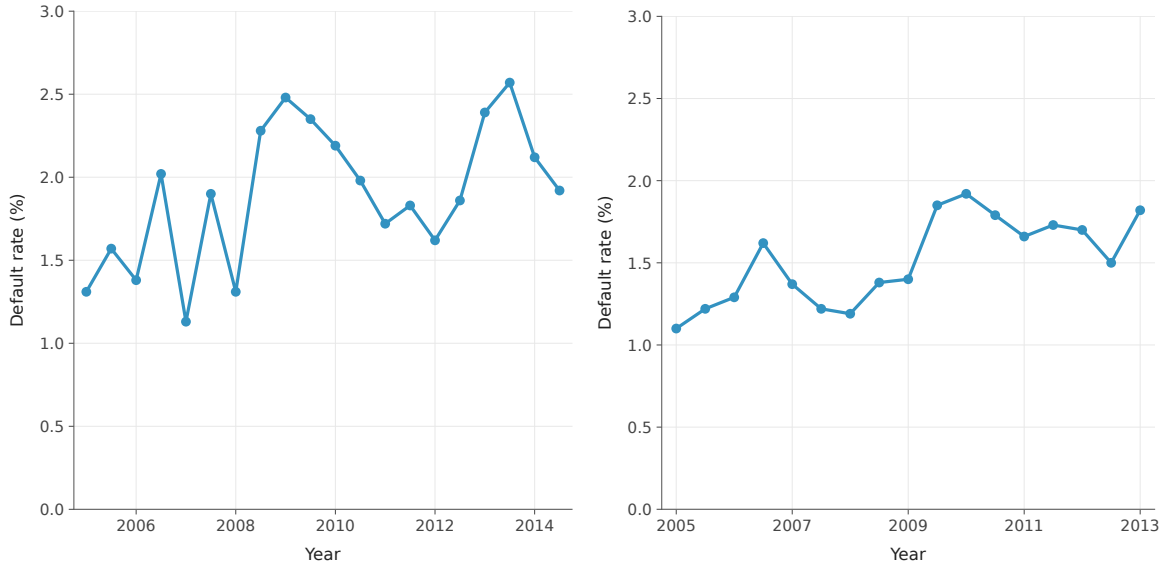
Finally, Figure B.5 reports the response of reported PDs to $\widehat{\alpha}_{bt}$. Panel (a) shows the average response, based on the $\{\widehat{\beta}_h\}_h$ estimates from the following regression

$$\widetilde{\mathbb{E}}_{bt+h} PD_{ft+h+4} - \widetilde{\mathbb{E}}_{bt-1} PD_{ft+3} = \gamma_{fth} + \xi_{bfh} + \beta_h \widehat{\alpha}_{bt} + u_{bfth}, \tag{B.7}$$

where $\gamma_{ft,h}$ and $\xi_{bf,h}$ are firm–quarter and bank–firm fixed effects.

Panel (b) plots the differential response for borrowers that rely more heavily on the reporting bank for credit. For each bank, we rank borrowers by the share of their total committed credit supplied by that bank and define D_{bft} as an indicator for borrowers in the top decile. We augment (B.7) with an interaction between $\widehat{\alpha}_{bt}$ and D_{bft}. Panel (b) plots the estimated interaction coefficients.

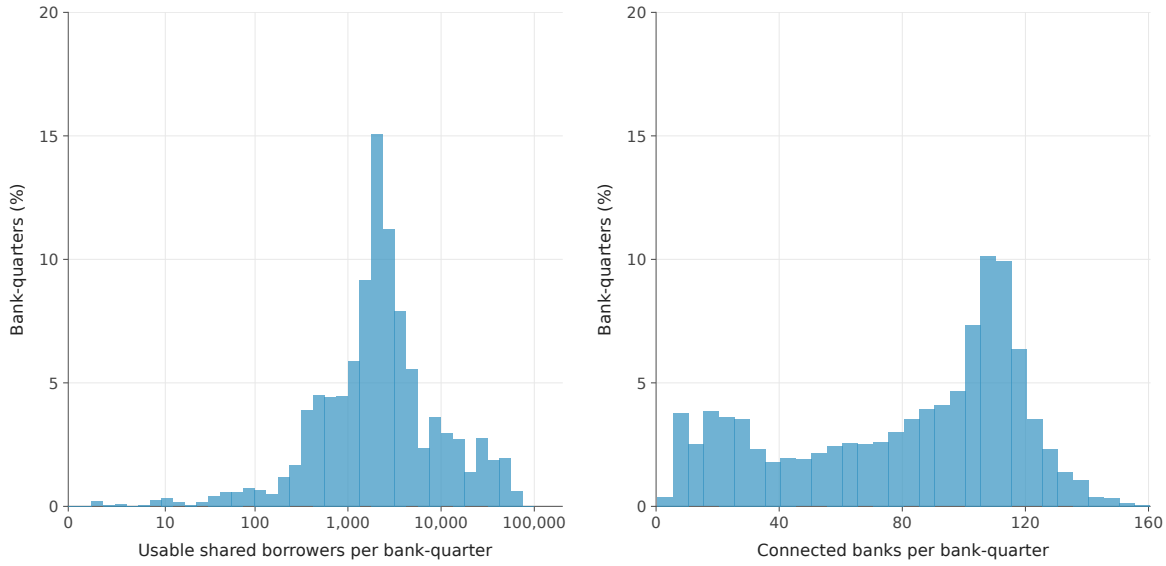
Figure B.1 – Aggregate variation in corporate default rates



(a) Germany **(b)** France

Notes: the figure plots realized default rates among corporate borrowers of commercial banks in Germany (panel (a)) and France (panel (b)). Default rates are annualized from semi-annual data. Source: Dietsch et al. (2016).

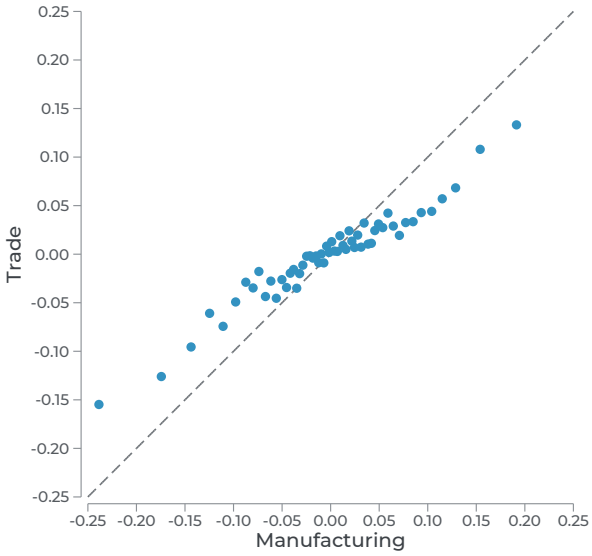
Figure B.2 – Number of shared borrowers and connected banks



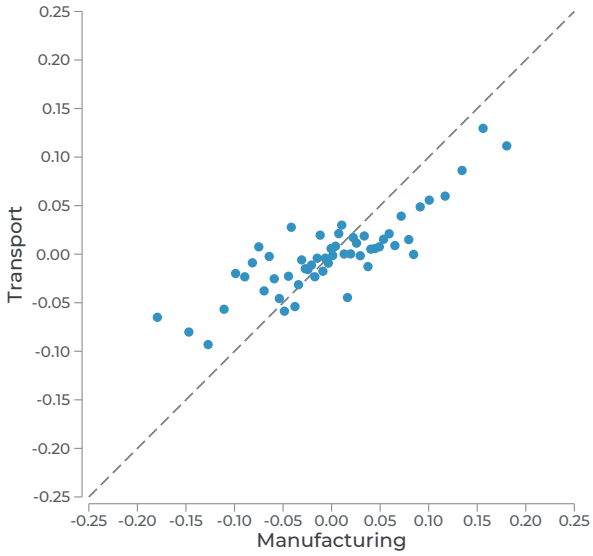
(a) Shared borrowers per bank-quarter **(b)** Connected banks per bank-quarter

Notes: panel (a) shows the distribution of the number of distinct usable borrowers that each bank shares with at least one other bank in the same quarter. Panel (b) shows, for each bank-quarter, how many other banks lend to at least one of the same usable borrowers. We define a borrower as usable when the quarterly change in PD is observed at both lenders. For both panels the unit of observation is a bank-quarter.

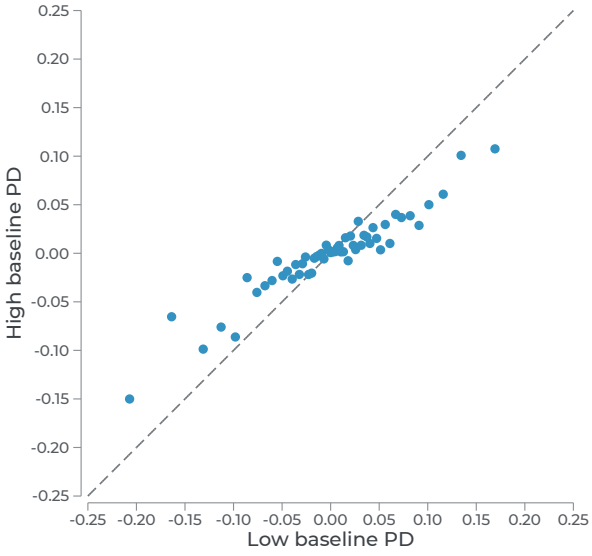
Figure B.3 – Stability of $\widehat{\alpha}_{bt}$ across Subsamples



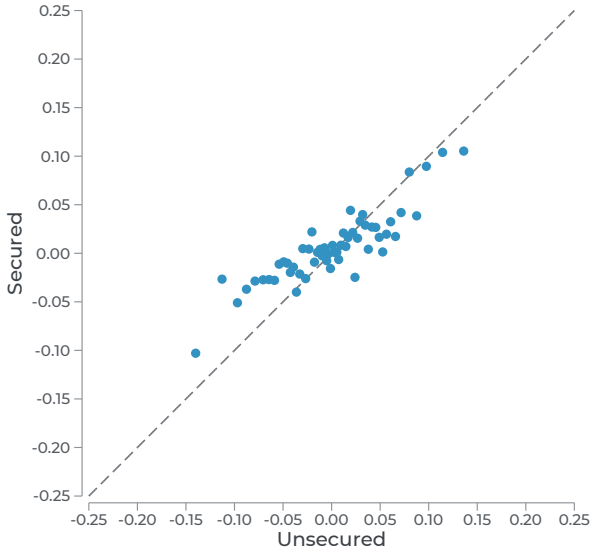
(a) Manufacturing vs trade subsamples



(b) Manufacturing vs transport subsamples



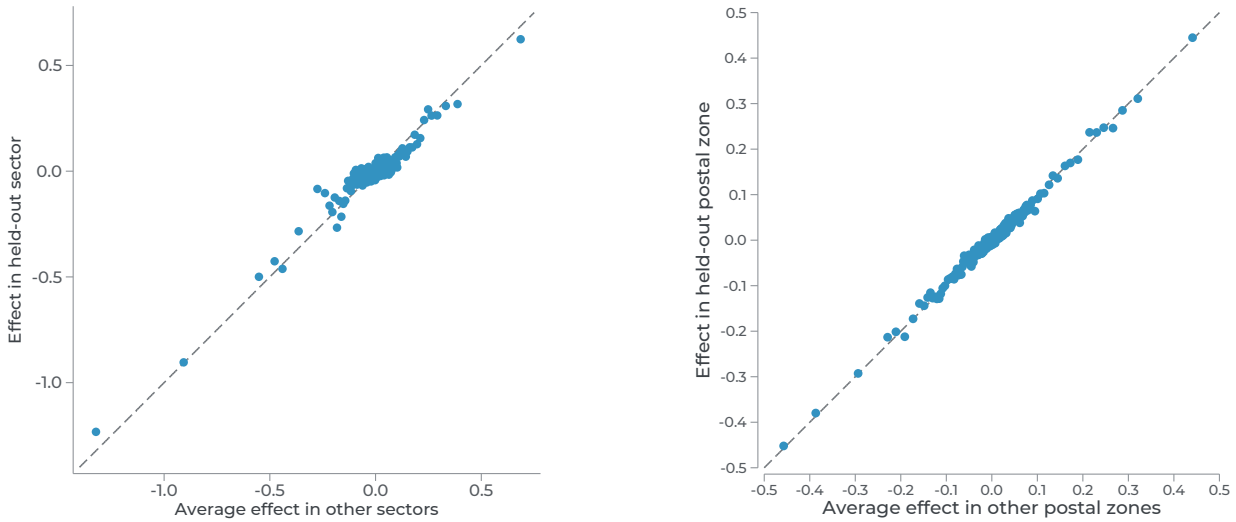
(c) Low vs high PD subsamples



(d) Secured vs unsecured subsamples

Notes: the figure compares $\widehat{\alpha}_{bt}$ obtained by estimating (21) on different borrower subsamples. Panels (a) and (b) compare estimates based on manufacturing borrowers with those based on borrowers in wholesale and retail trade and in transport, respectively. Panel (c) compares estimates based on borrowers with baseline PDs at or below the median with those based on borrowers above the median. Panel (d) compares estimates based on bank–firm relationships without collateral with those based on relationships with collateral.

Figure B.4 – Sectoral and Regional Heterogeneity in Expectation Shokcs

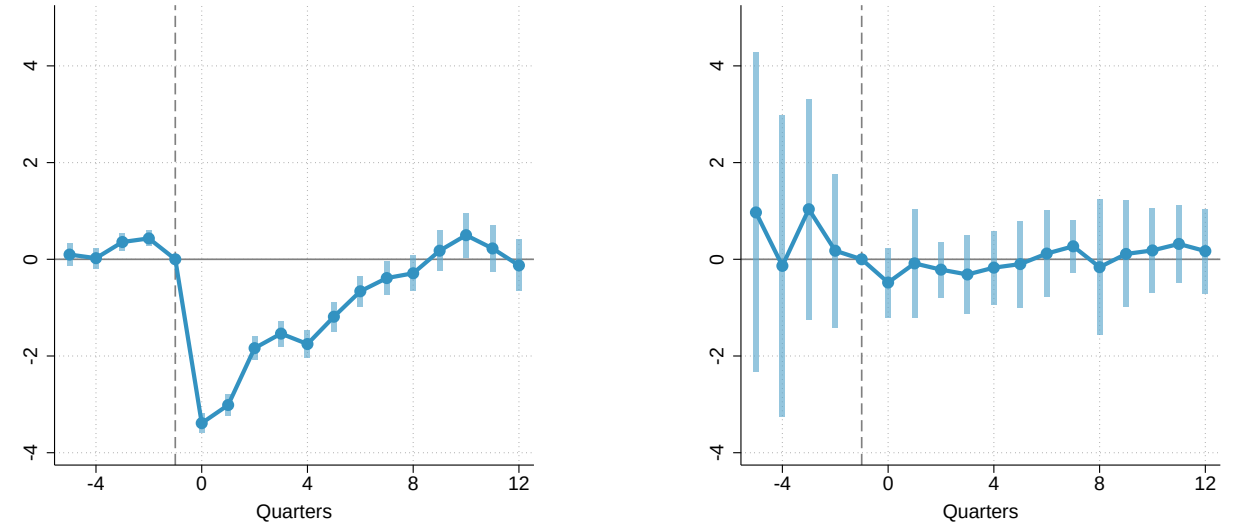


(a) Bank-sector-time fixed effects

(b) Bank-region-time fixed effects

Notes: the figure compares estimates obtained by applying (21), replacing bank–time fixed effects with bank–time–sector fixed effects in panel (a) and bank–time–region fixed effects in panel (b). Panel (a) plots a bank's estimated effect in one sector against the average of its effects in the other sectors in the same quarter. Panel (b) makes the analogous comparison across regions. The sector or region on the vertical axis is excluded from the horizontal-axis average.

Figure B.5 – Response of Reported PDs to Expectation Shocks



(a) Average response

(b) Heterogeneous response for most exposed borrowers

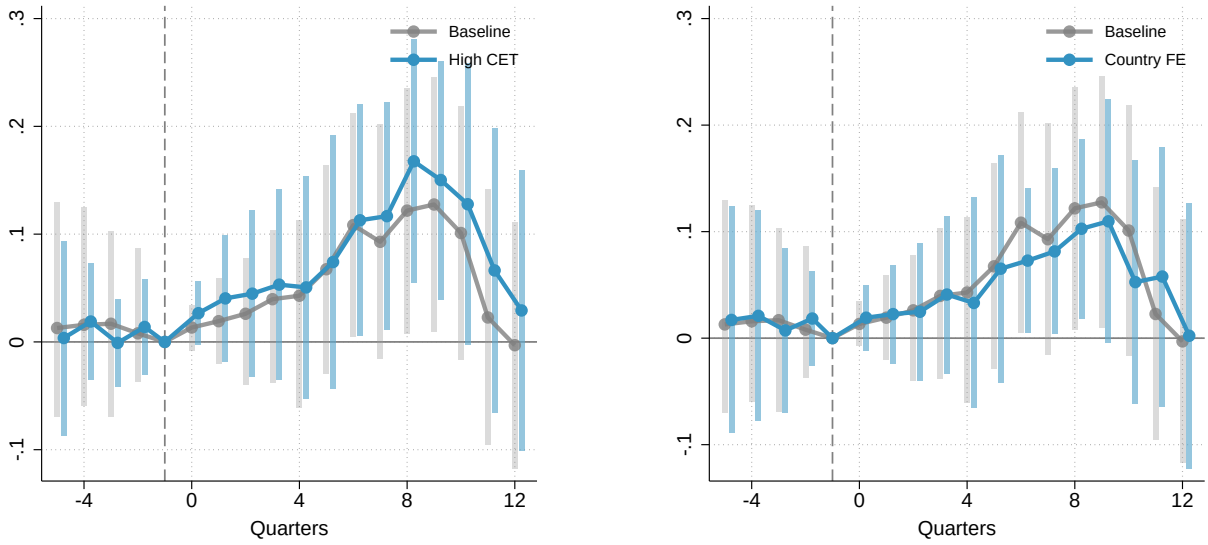
Notes: the figure reports local-projection responses of reported PDs to $\widehat{\alpha}_{bt}$. Panel (a) shows the average response. Panel (b) shows the coefficient on the interaction between the shock and an indicator for borrowers in the bank's top decile, ranked by the bank's share of their total committed credit. The vertical bars show 95% confidence intervals.

Robustness to Figure 6. Figures B.6 to B.8 report robustness checks for the results in panel (a) of Figure 6. Panel (a) of Figure B.6 shows that the estimates are unchanged when we restrict the sample to banks with above-median CET1 capital headroom. Panel (b) shows that the estimates remain qualitatively unchanged when we include country–time fixed effects in (22).

In Figure B.7, we re-estimate (22) excluding 2020 in panel (a) and restricting the sample to 2019Q3–2022Q4 in panel (b).

Finally, panels (a) and (b) of Figure B.8 show that the estimates are unchanged when we use a lag-augmented specification of (22) (Montiel Olea and Plagborg-Møller, 2021) or control for four lags of pre-shock leverage growth, respectively.

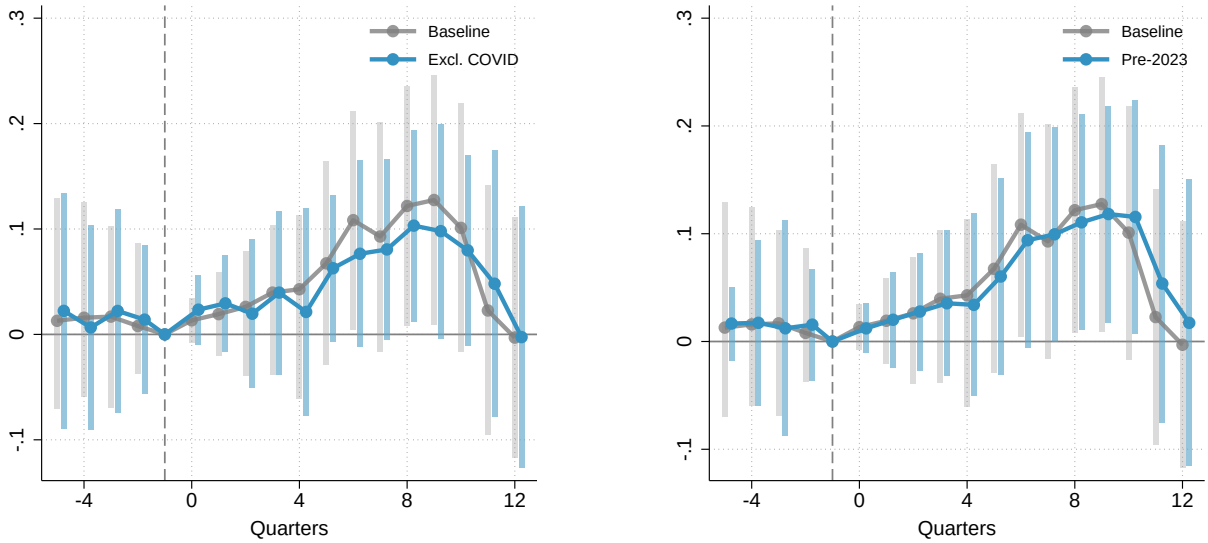
Figure B.6 – Robustness to high regulatory capital headroom and country-time fixed effects



(a) Top 50% of capital headroom subsample **(b)** Country-time fixed effects

Notes: panel (a) re-estimates the results from Figure 6 on the subsample of banks in the top 50% of the CET1 capital headroom above the regulatory requirement. Panel (b) re-estimates the results from Figure 6 including country-quarter fixed effects. Bars represent 95% confidence intervals, with standard errors clustered at the bank level.

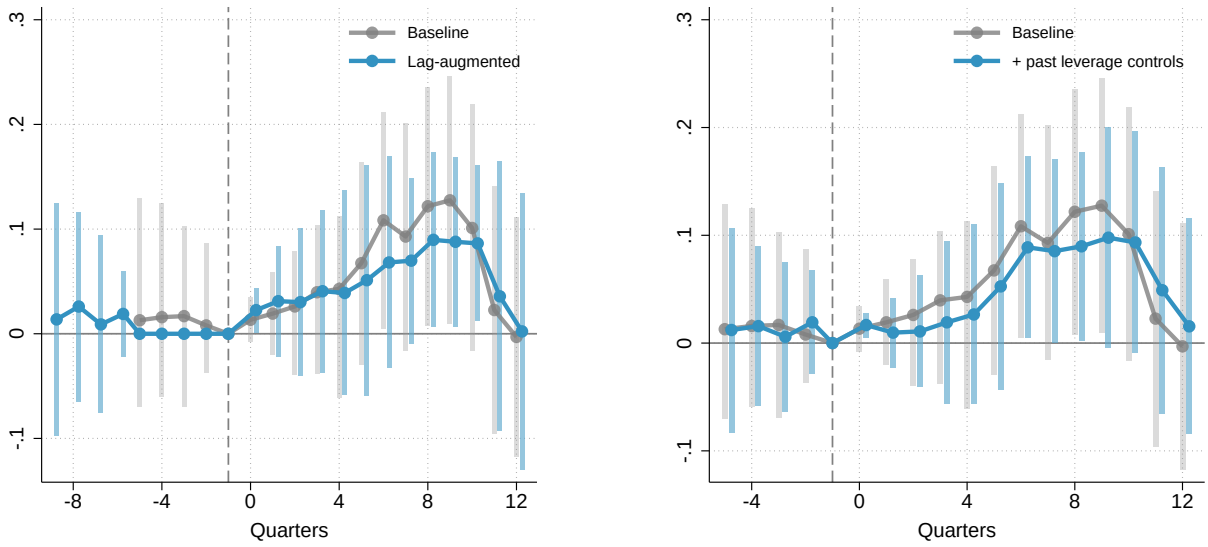
Figure B.7 – Robustness to different time samples



(a) Exclude 2020q1-2021q1 **(b)** 2019q3-2022q4 subsample

Notes: panel (a) re-estimates the results from Figure 6 excluding quarters between 2020q1 and 2021q1. Panel (b) re-estimates the results from Figure 6 on the 2019q3-2022q4 subsample. Bars represent 95% confidence intervals, with standard errors clustered at the bank level.

Figure B.8 – Robustness to lag-augmented specification and leverage controls

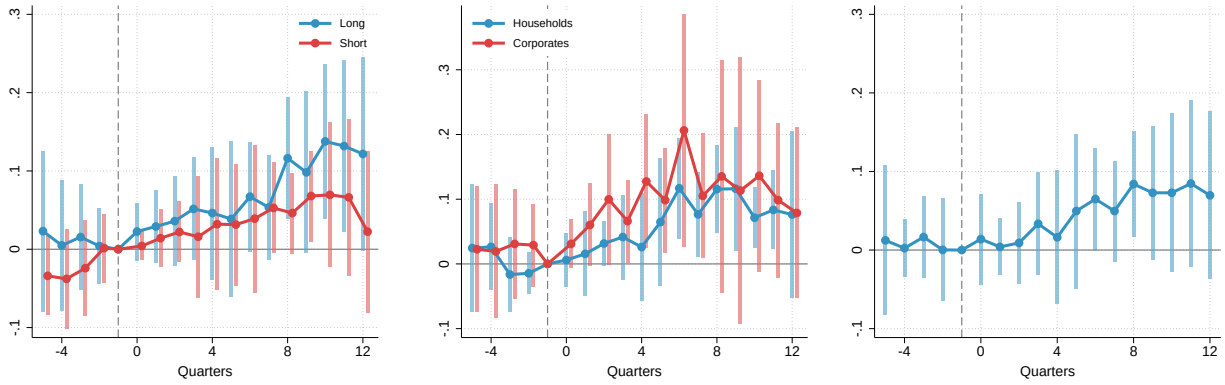


(a) Lag-augmented **(b)** Control for pre-shock leverage change

Notes: panel (a) re-estimates the results from Figure 6 using a lag-augmented local projection specification as in Montiel Olea and Plagborg-Møller (2021). Panel (b) re-estimates the results from Figure 6 controlling for four lags of quarterly leverage growth before the shock. Bars represent 95% confidence intervals, with standard errors clustered at the bank level.

Additional results. In Figure B.9, we re-estimate the baseline local projection (22) using alternative outcome variables. Panel (a) reports the responses of short- and long-term loans, defined as loans with original maturities below and above five years, respectively. Panel (b) reports the responses of loans to households, including mortgages and consumer credit, and to non-financial corporations. Finally, panel (c) reports the response of the loans-to-equity ratio. For these figures, we the shaded bars plot 90% confidence intervals, with standard errors clustered at the bank level.

Figure B.9 – Additional outcome variables



(a) Short vs long maturity

(b) Corporate vs household loans

(c) Loans-to-equity ratio

Notes: panel (a) estimates the same specification as in Figure 6 using the total stock of long and short maturity loans as outcome variables. We define short maturity loans as loans with original maturity of less than five years and long maturity loans as loans with original maturity of more than five years. Panel (b) uses total corporate and household loans as outcome variables. Household loans include mortgages and consumer credit. Panel (c) plots estimates from the same specification as in Figure 6 using the total loans to equity ratio as the outcome variable. Bars represent 90% confidence intervals, with standard errors clustered at the bank level.

B.3 Construction of Change in Risk-Weighted Assets

We now discuss how we construct the change in risk-weighted assets associated with the PD revisions, keeping loan exposures fixed at the pre-shock period. The regulatory capital requirement for the exposure bank b has with borrower f in period t is

$$C_{bft} = 1.06\,\text{EAD}_{bft}\,\text{LGD}_{bft}F(PD_{bft}, M_{bft}, S_{ft}),$$

where EAD is exposure at default in euros, LGD is the regulatory loss-given-default rate, M is the maturity of the exposure in years, and S denotes borrower's sales, Φ is the standard normal

cumulative distribution function, and:

$$F(PD, M, S) = \left[\Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{R(PD,S)}\,\Phi^{-1}(0.999)}{\sqrt{1-R(PD,S)}}\right) - PD\right]\frac{1+(M-2.5)B(PD)}{1-1.5B(PD)},$$

and

$$R(PD,S) = 0.24 - 0.12\frac{1-e^{-50PD}}{1-e^{-50}} - 0.04\frac{50-\bar{S}}{45}, \qquad \bar{S} = \min\{50, \max\{5, S\}\},$$
$$B(PD) = \left(0.11852 - 0.05478\ln PD\right)^2.$$

Because in our data we do not observe exposure at default in euros and loss-given-default we proxy EAD with the outstanding loan balance in $t-1$, L_{bft-1} and impose a common LGD of 21.5%. We proxy effective maturity with remaining maturity. For borrowers with multiple loans, we use the loan-balance-weighted average.

Then, the percentage change in capital requirements associated with borrower f due to PD revision, holding all other inputs fixed, is:

$$\Delta\widehat{C}_{bft} = \left[\frac{F(PD_{bft}, M_{bft-1}, S_{ft-1})}{F(PD_{bft-1}, M_{bft-1}, S_{ft-1})} - 1\right].$$

We then construct the bank-level change in capital requirements for bank b as

$$\Delta\widehat{C}_{bt} = \left[\frac{\sum_f L_{bft-1}F(PD_{bft}, M_{bft-1}, S_{ft-1})}{\sum_f L_{bft-1}F(PD_{bft-1}, M_{bft-1}, S_{ft-1})} - 1\right] \tag{B.8}$$

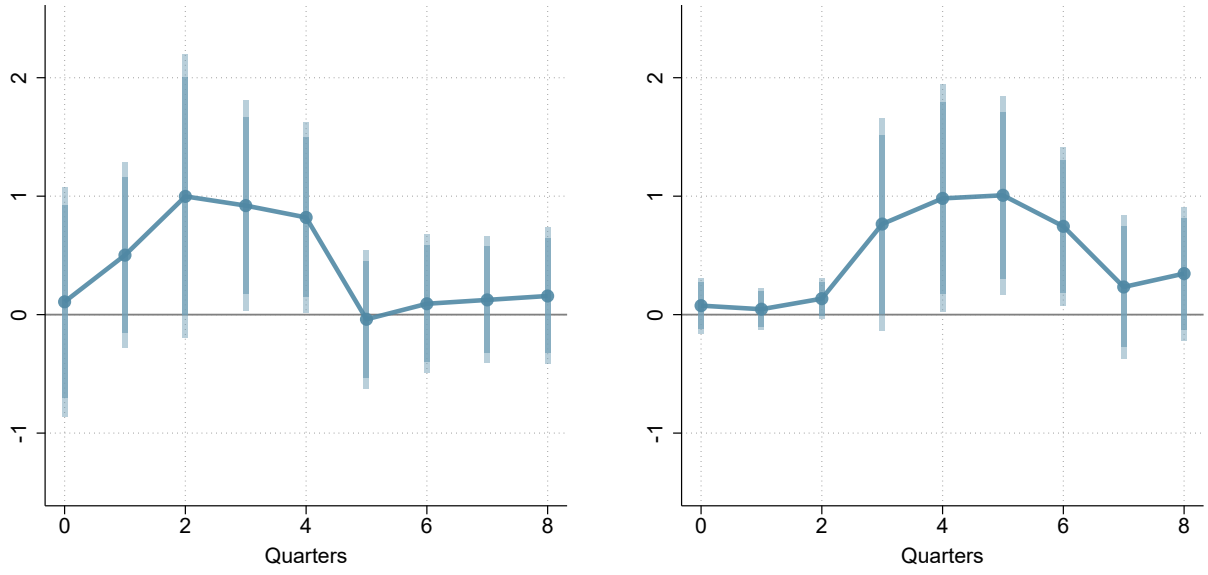
and include (B.8) above as a control in the right hand side of (22).

C Appendix to Section 4

In this Section we report robustness checks and additional results for the exercises in Section 4.

Robustness to Figure 7. Figure C.1 reports robustness exercises for the response of banks' default forecast error to the Angeletos et al. (2020) main business cycle shock from Figure 7. Panel (a) reports the estimates of the forecast error response to the Angeletos et al. (2020) (based on equation (28)) in the case in which we don't control for lagged aggregate variables. Panel (b) instead re-estimates (28) using the Känzig (2021) oil news shock as the aggregate shock measure.

Figure C.1 – Robustness for the forecast error response



(a) No lagged controls **(b)** Känzig (2021) oil shock

Notes: panel (a) plots the estimates for $\{\widehat{\beta}_h\}_h$ from (28) in the case in which we do not control for lagged aggregate variables. Panel (b) the estimates for $\{\widehat{\beta}_h\}_h$ from (28) for the Känzig (2021) oil news shock. Light and dark bands represent 90% and 95% confidence intervals, with errors clustered at the bank level.

Forecast error on forecast revision regressions. Table C.1 presents complementary evidence of overreaction of banks' expectations, by showing estimates of regressions of default forecast errors on forecast revisions (Coibion and Gorodnichenko, 2015, Bordalo et al., 2020).

We define the forecast error bank b makes in assessing firm f at time t as in equation (26) in

Table C.1 – Forecast Errors and Forecast Revisions

	(1)	(2)	(3)
Panel A. Individual bank–firm forecasts			
FR_{bft}	-0.5002^{***}	-0.4501^{***}	-0.4550^{***}
	(0.0012)	(0.0007)	(0.0007)
Observations	24,900,770	24,664,267	24,664,081
Bank–firm fixed effects		✓	✓
Quarter fixed effects		✓	✓
Bank–quarter fixed effects			✓
Panel B. Firm-level consensus forecasts			
$\overline{FR}_{ft}$	-0.5285^{***}	-0.5324^{***}	-0.4196^{***}
	(0.0023)	(0.0022)	(0.0012)
Observations	9,857,566	9,857,566	9,785,661
Firm fixed effects			✓
Quarter fixed effects		✓	✓

Notes: the table reports regressions of one-year ahead default forecast errors on quarterly PD revisions. The forecast error is defined as in (C.1), as the difference between an indicator variable for firm default within the next 12 months and the current reported one-year ahead PD. The revision is defined as in (C.2), as the change in the PD relative to last quarter. Both variables are expressed in percentage points. Estimates in panel A are at the individual bank–firm level. Estimates in panel B are at the consensus level, based on equal-weight averages across a firm's lenders. Standard errors clustered by firm in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

the main text:

$$\mathcal{E}_{bft} = PD^{e}_{bft} - PD^{e}_{bft-1}, \tag{C.1}$$

and the forecast revision between $t-1$ and t as

$$FR_{bft} = PD^{e}_{bft} - PD^{e}_{bft-1}. \tag{C.2}$$

We then define the same two variables at the consensus level, $\overline{\mathcal{E}}_{ft}$ and $\overline{FR}_{ft}$ as the simple average of the forecast error and forecast revision among all of the lenders of firm f in period t.

Panel A of Table C.1 reports estimates of regressions of $\mathcal{E}_{bft}$ on FR_{bft} without controls as well as controlling for bank–firm and quarter fixed effects as well as bank–firm and bank–quarter fixed effects. Panel B reports estimates of regressions at the consensus level, wihtout controls as well as controlling for quarter fixed effects and quarter and firm fixed effects. We cluster standard errors at the firm level.

D Appendix to Section 5

D.1 Calibration of Idiosyncratic Return Risk

We calibrate idiosyncratic return risk using banks' financial statements. Combining income-statement flows with balance-sheet exposures allows us to construct an accounting measure of loan-portfolio returns that incorporates both interest income and credit losses.

Let I^L_{bt} denote interest income on loans, C^L_{bt} denote realized loan losses net of recoveries, and L_{bt} denote the gross outstanding loan stock. We construct

$$r^L_{bt} = \frac{I^L_{bt} - C^L_{bt}}{L_{b,t-1}}.$$

Income and loss flows refer to the same reporting period, portfolio, and level of consolidation. Net credit losses are measured using write-offs less recoveries. The resulting measure captures the return on lending before funding costs, operating expenses, and taxes. Measuring returns relative to loans is appropriate because σ_ε governs risk per unit of intermediary investment; leverage then determines how this risk translates into fluctuations in net worth.

We then estimate the following fixed effect regression on bank returns, in line with the iid process for returns we use in our model:

$$r^L_{bt} = \alpha_b + \delta_t + u_{bt},$$

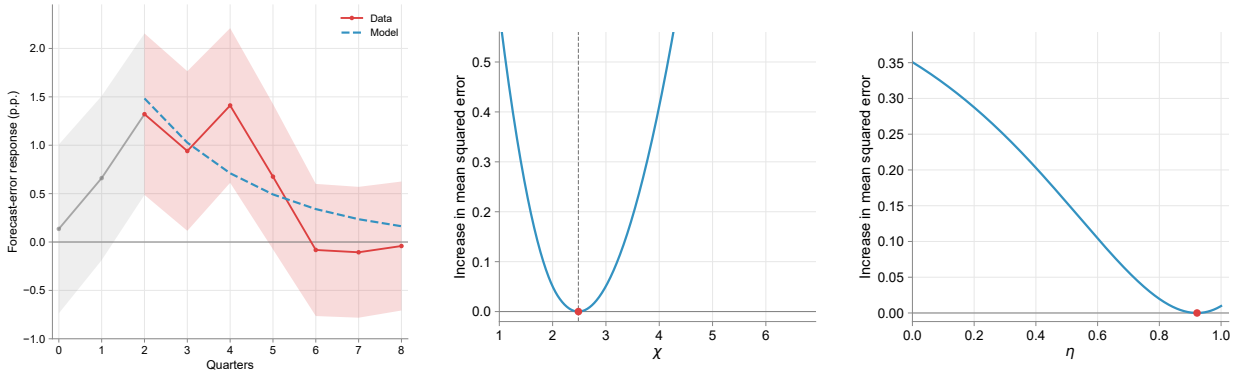
where α_b denotes a bank fixed effect, absorbing persistent differences in average returns, and δ_t is a time fixed effect, absorbing movements in returns common across banks. We then set σ_ε equal to the standard deviation of the residuals u_{bt} from this regression, run at quarterly frequency.

D.2 Calibration of Expectation Formation

We calibrate the two parameters χ and η governing the expectation formation process using the results from Figure 7.

First, we fit an AR(1) to the response of realized defaults in panel (a), getting a quarterly persistence of the default IRF of 0.7505. Using the expectation formation process (29) we then simulate the path of forecast errors following an innovation that raises the target variable x_t by 1 percentage point on impact and with the same persistence of the estimated realized default persistence from panel (a) of Figure 7. We then choose χ and η jointly to minimize the mean

Figure D.1 – Calibration of Expectation Formation Parameters



(a) IRF matching **(b)** Loss function along χ **(c)** Loss function along η

Notes: Panel (a) compares the empirical forecast-error response (solid red) with the fitted model response (dashed blue), in percentage points. Shading areas denote 90% confidence intervals of the empirical forecast error response. Horizons 0 and 1 of the empirical response are grayed out as they are excluded from the calibration, horizon 2 of the empirical forecast error response corresponds to horizon zero in the model. Panels (b) and (c) plot the increase in mean squared error relative to its minimum over χ and η, respectively. At each point, the other behavioral parameter is reoptimized. Red dots denote the calibrated values.

squared distance between the model-implied path of forecast errors and the forecast error response from panel (b) of Figure 7. Because our model is not well-suited to generate hump-shaped forecast error dynamics, we fit the forecast error response over horizons $h = 2 - 8$.[50] The resulting parameter values are $\chi = 2.484$ and $\eta = 0.922$. Panel (a) of Figure D.1 plots the empirical forecast error response together with the dynamics of the forecast error generated by the calibrated expectation formation process we use in our model. Panels (b) and (c) show the increase in the mean squared error relative to the calibrated values over χ and η, respectively.

D.3 Solution Algorithm

We now explain the algorithm we use to solve the non-FIRE transition of our heterogeneous intermediary model. Throughout, we truncate the solution horizon at $T = 250$ quarters and use the stationary equilibrium to set continuation values and prices beyond T.

Input: Initial aggregate state, inherited intermediary portfolios (to compute revaluation effects) and net-worth distribution; actual productivity path $\{A_t\}_{t=0}^{T-1}$; expectation-formation rule; and convergence tolerance ε.

Output: Equilibrium paths for prices, aggregate quantities, intermediary policies, and net-worth distributions.

[50] We align the model so that horizon zero in the model is fitted to the horizon two forecast error response from the data.

1. **Initialization.** Construct the matrix of perceived productivity paths,

$$\{A^e_{t,s}\}_{0\le t\le s<T}, \qquad A^e_{t,t} = A_t,$$

conditional on no further aggregate innovations.

2. **Initial guess.** Guess paths for capital prices, real interest rates, real wages, together with household consumption in the final period.

3. **Repeat until convergence:**

   (a) **Recover aggregate quantities and prices.** Starting from the inherited capital stock K_0, use the capital-goods producer's optimality condition and capital accumulation:

$$q_t = 1 + \Psi_I(I^S_t, K_t), \qquad K^S_{t+1} = (1-\delta)K_t + I^S_t.$$

   Recover consumption backward from the guessed terminal value using

$$C_t = \frac{C_{t+1}}{[\beta_{hh}(1+r_{t+1})]^{1/\gamma_{hh}}}.$$

   Use the production and labor-demand equations to obtain employment, output, and rental rates. We back out wage inflation by solving the New Keynesian Phillips curve backward, and recover price inflation and nominal interest rates combining the definition of real-wage growth and the Fisher equation.

   (b) **Revaluation.** Construct the initial condition for the post-shock distribution G_0 by applying the net-worth accumulation equation to inherited portfolios at the newly realized prices and returns.

   (c) **Solve the intermediary block.** For each date $t = 0, \dots, T-1$:

   - **Subjective price forecasts.** Conditional on an expected path of future productivity, use the FIRE model to recover subjective prices and returns through the aggregate equations, conditional on the current observed aggregate capital and distribution of intermediaries' net worth.
   - **Solve policies backward.** We solve the intermediary's Bellman equation by backward induction plugging in the path of subjective beliefs about future prices and returns to obtain the policy functions for period t

$$d_t(n), \qquad k'_t(n), \qquad a_t(n) = n - d_t(n).$$

- **Aggregation.** Compute aggregate capital demand, investment demand, and aggregate dividends by integrating policy functions:

$$K_{t+1}^D = \int k_t'(n)\, dG_t(n), \qquad I_t^D = K_{t+1}^D - (1-\delta)K_t,$$

$$D_t = \int d_t(n)\, dG_t(n).$$

- **Update next-period pre-shock distribution.** We simply apply the definition of the law of motion for individual intermediarie's net worth using the obtained policies and the realized prices and returns:

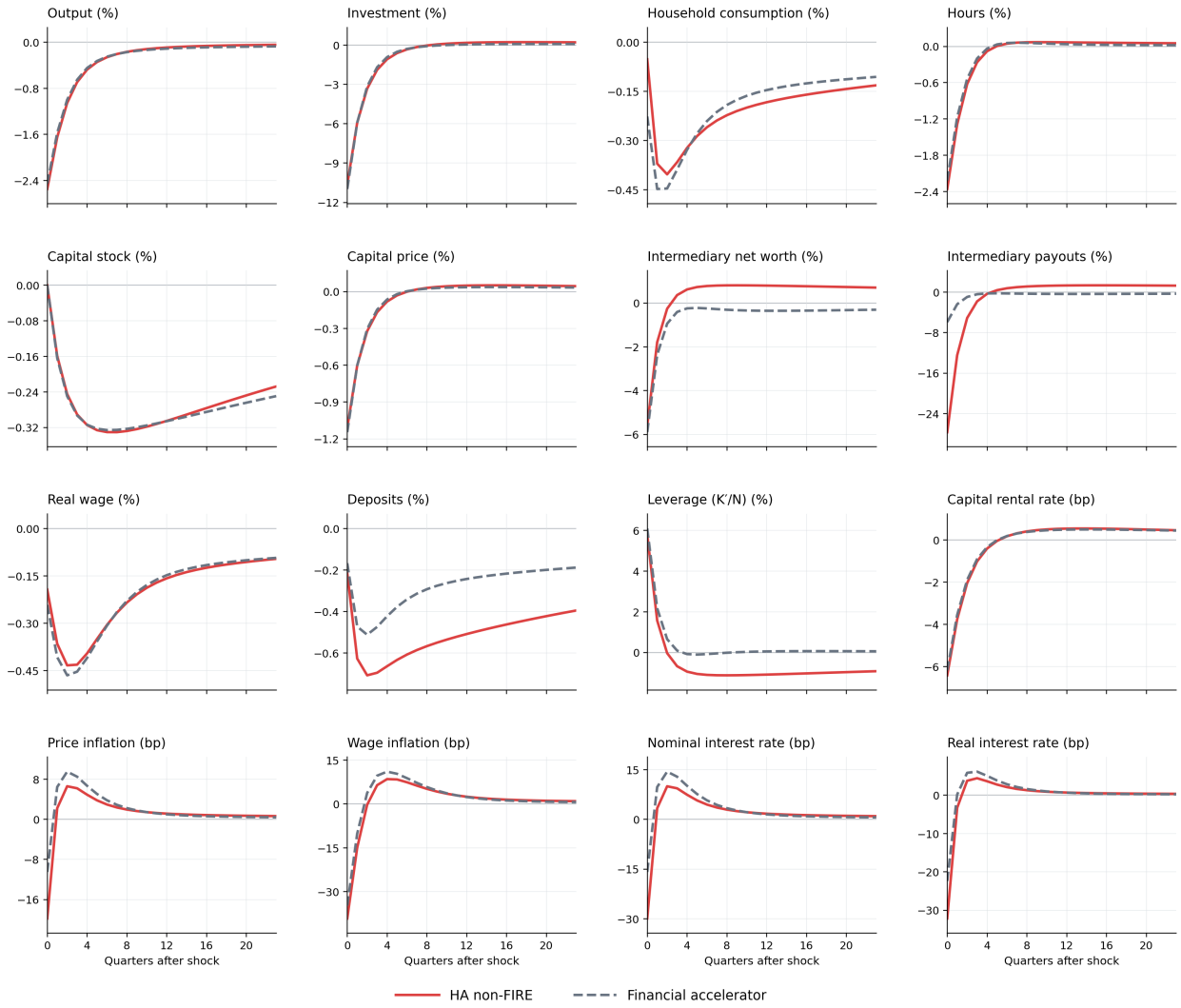
$$G_{t+1} = \Gamma\big(G_t, d_t, k_t';\ \text{realized prices and returns}\big)\,,$$

where Γ integrates over the idiosyncratic return shock and incorporates both exogenous and endogenous exit and mechanical entry to preserve the mass of intermediaries.

(d) **Evaluate equilibrium residuals.** We repeat this until the goods-market clearing, capital-market clearing, and Taylor-rule residuals are below convergence.

4. **Repeated innovations.** For experiments with repeated shocks, we apply this algorithm sequentially, implementing only current-period policies and treating each new innovation as unanticipated. We bring forward aggregate capital, the net-worth distribution, intermediary portfolios, and all other predetermined states, including intermediaries' memory that enters their forecasts of future aggregate productivity.

Figure D.2 – Additional Model IRFs

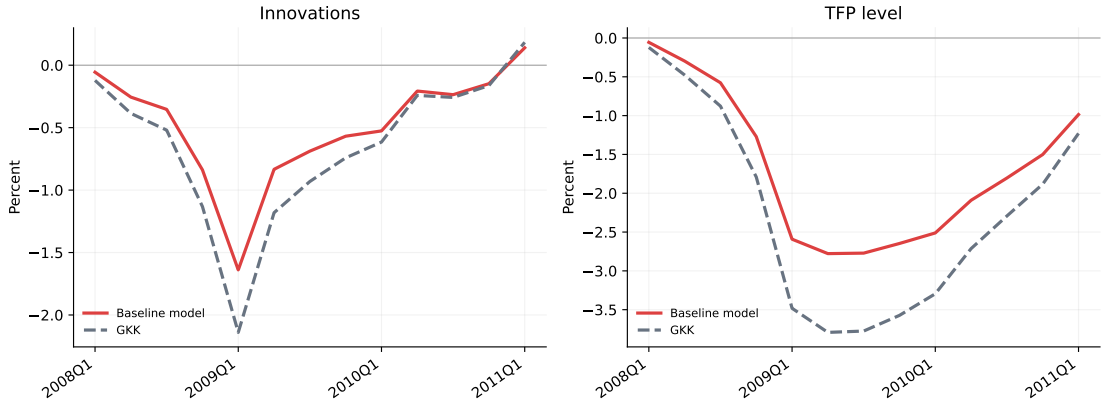


Notes: IRFs for additional model variables in our heterogeneous-intermediary model with overreaction and the financial accelerator benchmark.

D.4 Global Financial Crisis inputs

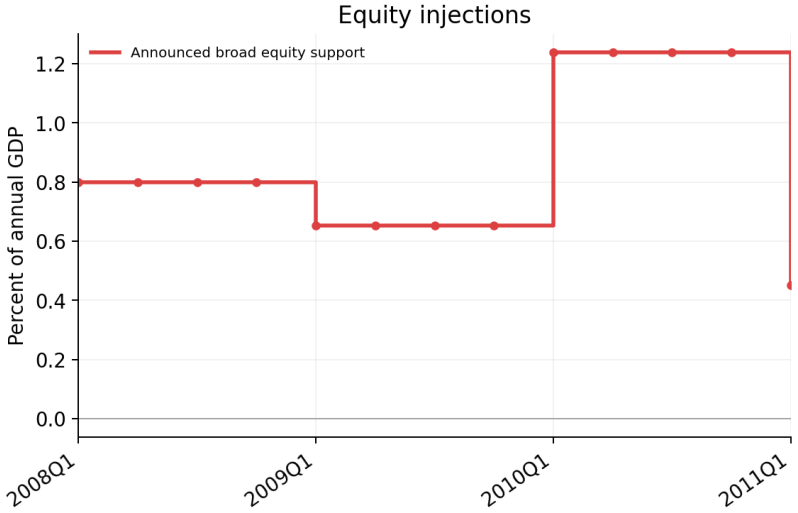
Figure D.3 reports the model-specific TFP innovations fitted to reproduce the observed output path, together with the implied TFP levels. Figure D.4 reports the path of recapitalizations (as a share of GDP) used in the exercise.

Figure D.3 – Path for Aggregate Productivity Shocks in the Global Financial Crisis exercise



Notes: quarterly TFP innovations (left) and implied TFP levels (right), in percent. Innovations are fitted separately in our heterogeneous agent model and the financial accelerator benchmark to reproduce observed output conditional on the path of recapitalizations.

Figure D.4 – Path for Recapitalizations in the Global Financial Crisis exercise

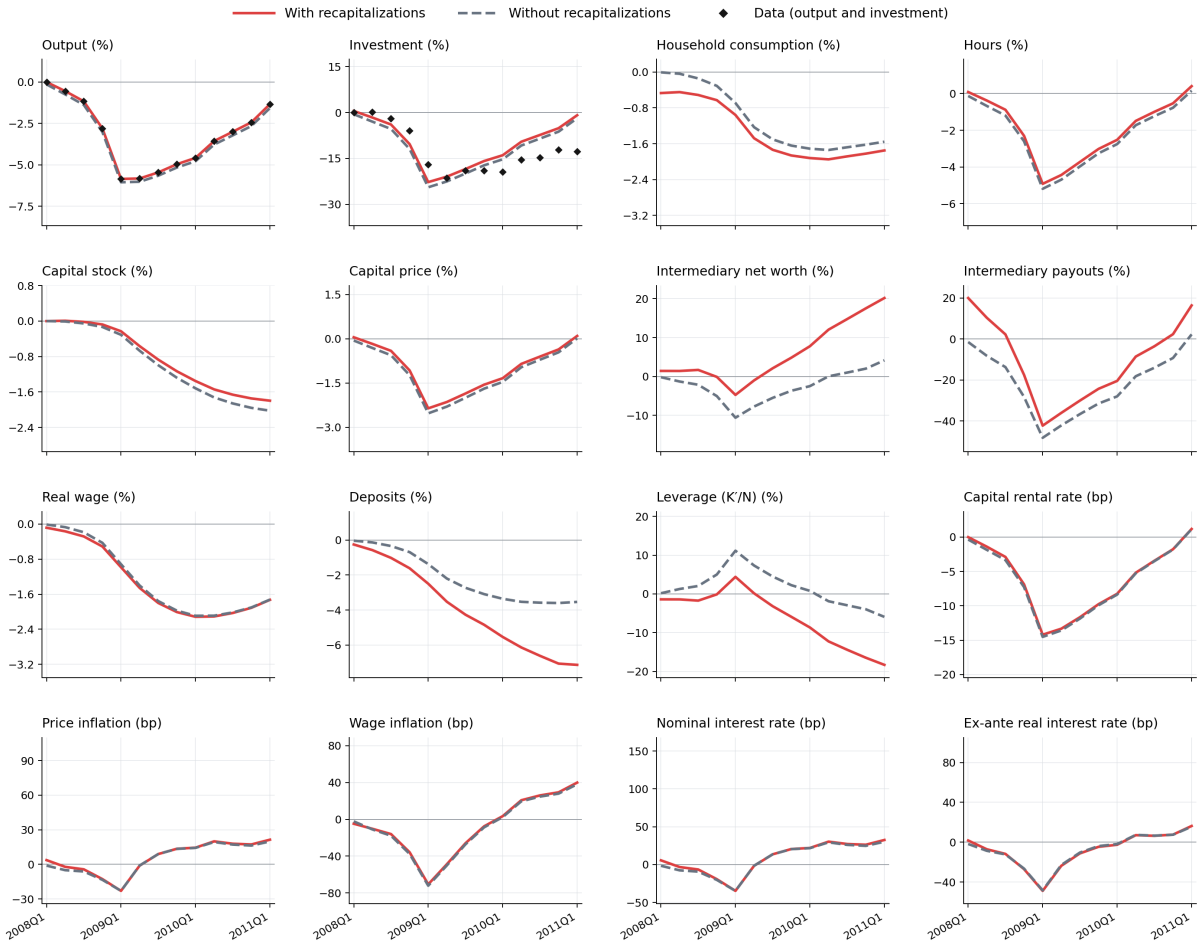


Notes: path for recapitalizations used in Figure 11 as a percent of annual GDP.

D.5 Additional variables in the Global Financial Crisis exercise

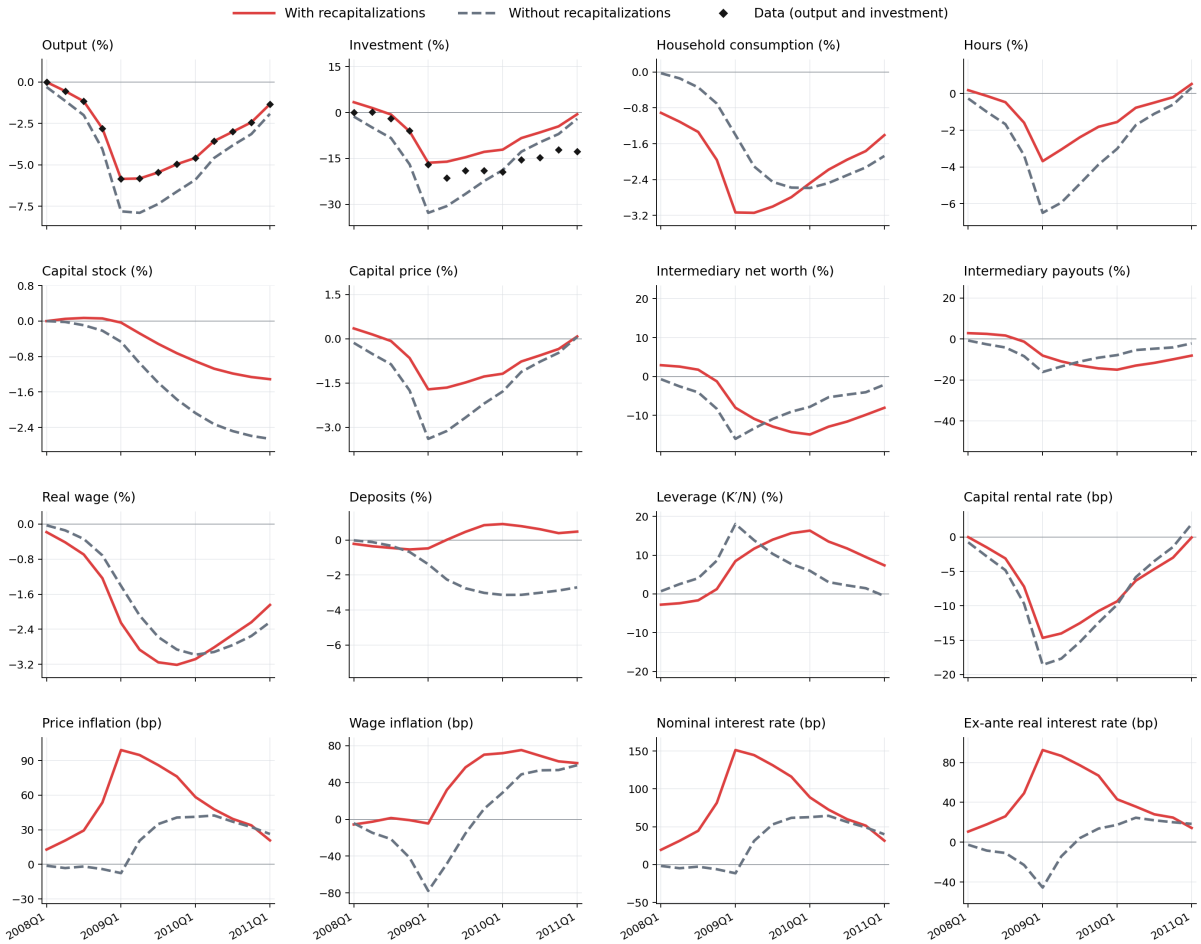
Figures D.5 to D.7 extend the comparisons in Figure 11 to a broader set of macroeconomic and financial variables. Each model retains its own fitted productivity innovations across policy counterfactuals. Levels and leverage are expressed as percentage deviations from steady state; interest rates and inflation are quarterly basis-point deviations, without annualization. Capital is the inherited stock, leverage is chosen physical capital divided by pre-payout net worth, and the real interest rate is the ex-ante rate contracted for the following quarter. All three figures use identical scales.

Figure D.5 – Global Financial Crisis: additional variables in the baseline model



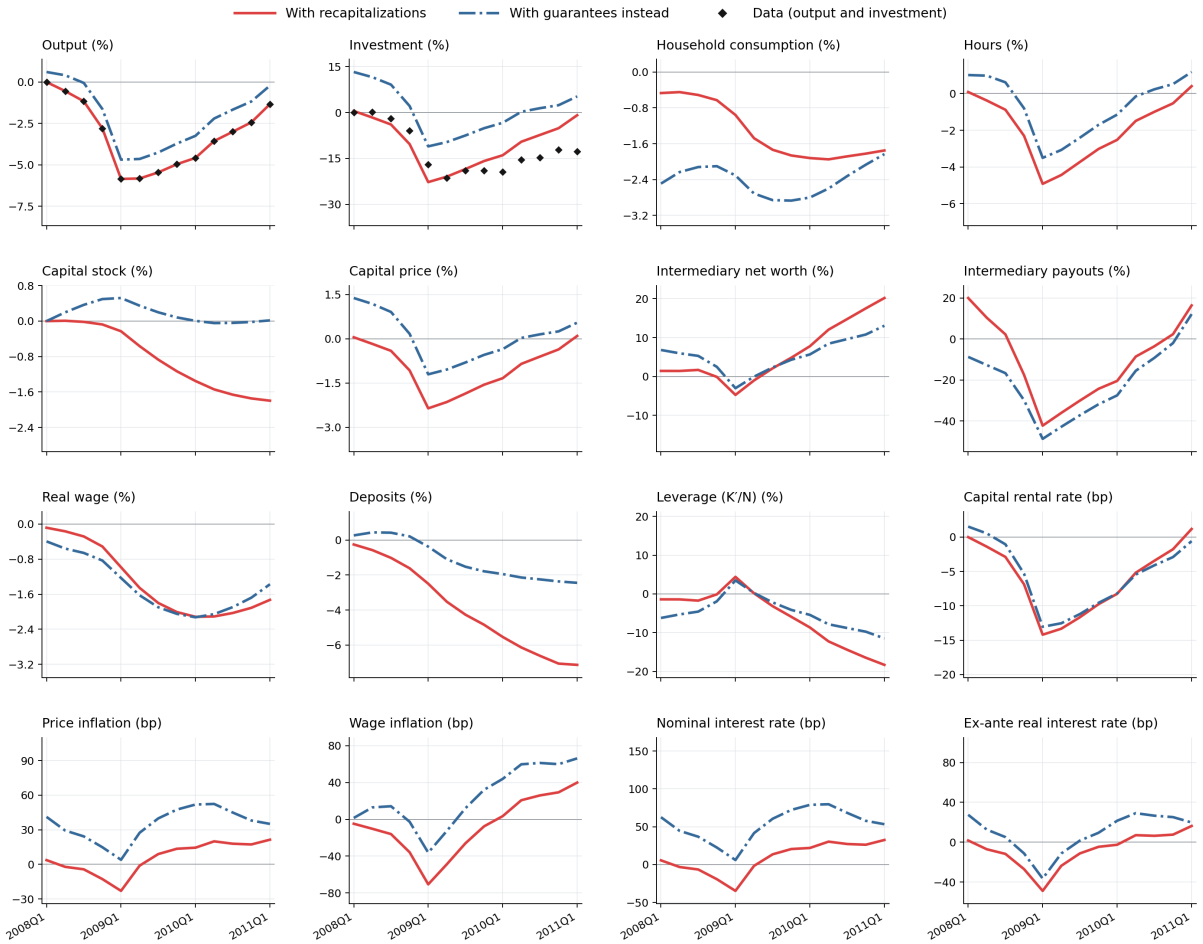
Notes: simulated path for additional variables for both our heterogeneous-intermediary model with overreaction, with recapitalizations (solid red) and without them (dashed gray), over 2008Q1–2011Q1. Black diamonds denote observed output and investment. Output is targeted under recapitalizations; investment is untargeted. Model paths are relative to steady state.

Figure D.6 – Global Financial Crisis: additional variables in the financial accelerator



Notes: simulated path for additional variables for the financial-accelerator benchmark with recapitalizations (solid red) and without them (dashed gray), over 2008Q1–2011Q1. Black diamonds denote observed output and investment. Output is targeted under recapitalizations; investment is untargeted. Variable definitions and units are as in Figure D.5.

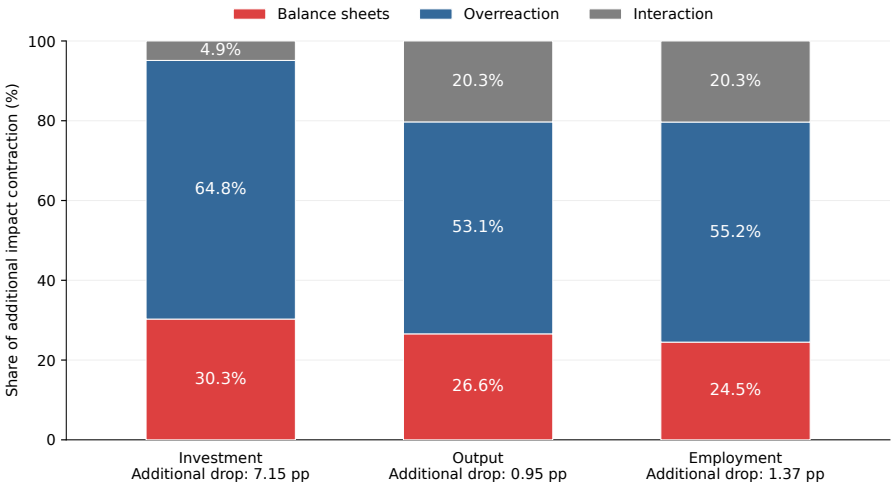
Figure D.7 – Global Financial Crisis: recapitalizations versus guarantees in the baseline model



Notes: simulated path for additional variables for the heterogeneous-intermediary model with overreaction, with recapitalizations (solid red) or loan guarantees instead (blue dash-dotted), retaining the productivity innovations fitted under recapitalizations. Variable definitions and units are as in Figure D.5.

D.6 Decomposing boom–bust amplification

Figure D.8 – Expectations, balance sheets, and boom–bust amplification



Notes: decomposition of the additional impact contraction after the 28-quarter non-FIRE boom, relative to a reference without overreaction of expectations and with intermediaries' inherited balance sheets and the aggregate capital stock reset to steady state. Expectations are blue, inherited balance sheets red, and the interaction gray. Bars show percentage shares of the additional contraction. Total additional contractions below the bars are in pp of steady-state investment or output; positive values indicate a larger contraction.