# A Distributional Theory of Household Sentiment

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*This paper: Aiyagari meets Diagnostic Expectations*

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II

III

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- Develop new tool to handle deviations from RE in HA models
- Sentiment distorts Euler Equation $\longrightarrow$ state/time dependent discount rate
- Sentiment jumps after income shocks $\longrightarrow$ consumption overreacts to income shocks

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- Positive income shock $\longrightarrow$ overoptimism $\longrightarrow$ under-saving $\longrightarrow$ poverty trap
- Large and distributed welfare cost

# PLAN FOR TODAY

I Empirical motivation

II Model and Methodology

III Theoretical Results

# SUGGESTIVE EVIDENCE ON EXPECTATIONS

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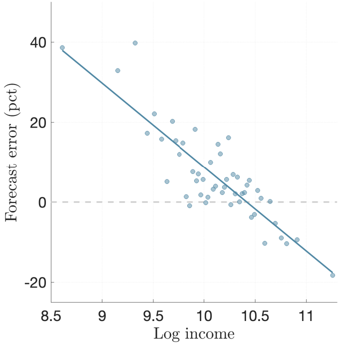

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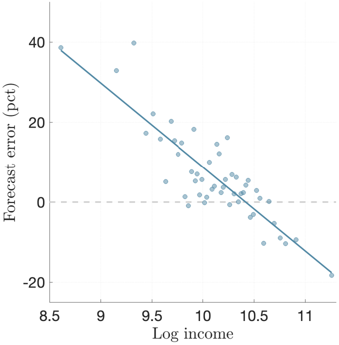

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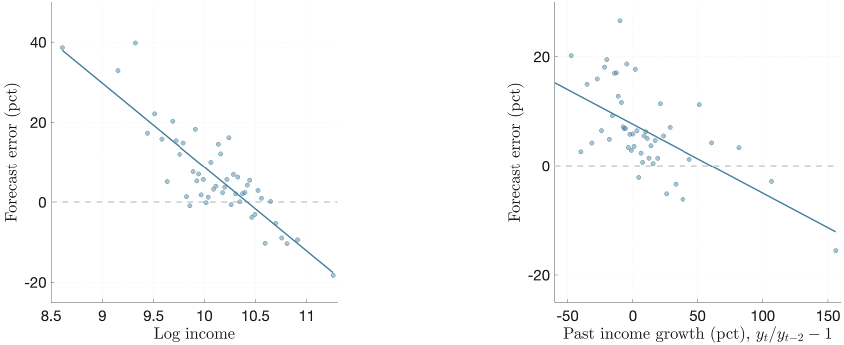
  - II Sentiment correlates with idiosyncratic income growth $y_t/y_{t-2} - 1$



# Model & Methodology

- Expectations biased by recent income shocks

$$\underbrace{dy_t}_{\text{log-inc. change}} = \underbrace{-\mu y_t dt}_{\text{drift}} + \underbrace{dN_t}_{\text{jump shocks}} \quad v.s. \quad \widetilde{dy_t} = \Big( - \mu y_t + \mathcal{S}_t \Big) dt + dN_t$$

Sentiment $\mathcal{S}_t \equiv \boldsymbol{\theta} \int_{-\infty}^{t} e^{-\boldsymbol{\kappa}(t-s)} dN_s \quad \longleftarrow$ discounted sum of shocks

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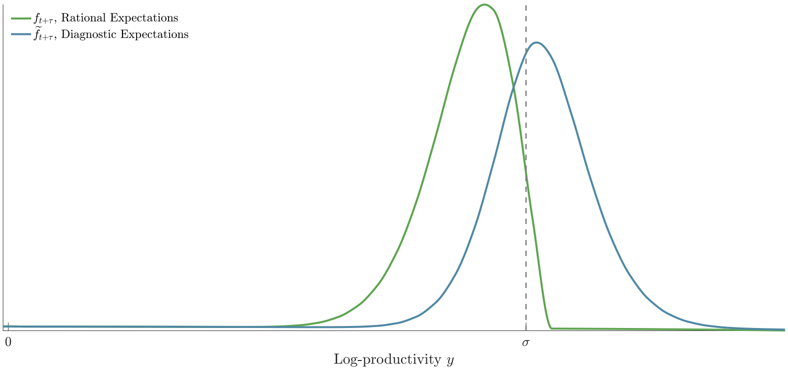

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- Embed within an incomplete market environment

$$\max_{\{c_t\}_{t\geq 0}} \quad \widetilde{\mathbb{E}}_{\mathbf{0}} \int_0^\infty e^{-\rho t} u(c_t) dt, \quad s.t. \quad \dot{a}_t = r a_t + e^{y_t} - c_t, \quad a \geq \underline{a}$$

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- Not trivial: household's perceived evolution of states $\neq$ true evolution of states

- Three states $x = (a, y, \mathcal{S})$: wealth $a$, log-productivity $y$ and sentiment $\mathcal{S}$:

$$V(a_0, y_0, \mathcal{S}_0) = \max_{\{c_t\}_{t\geq 0}} \quad \widetilde{\mathbb{E}}_{\mathbf{0}} \int_0^\infty e^{-\rho t} u(c_t) dt, \quad s.t. \quad \dot{a}_t = r a_t + e^{y_t} - c_t, \quad a \geq \underline{a}$$

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- Mean field game with rationality wedge

$$\begin{aligned} \rho V(x) &= \max_c \; u(c) + s(x, c)\partial_a V(x) + \mathcal{B} V(x) + \underbrace{\mathcal{S}\partial_y}_{\Psi} V(x) \\ 0 &= -\partial_a (s(x) g(x)) + \mathcal{B}^* g(x) \end{aligned}$$

# Results

## Proposition

$$\mathbb{E}_t \quad \frac{du'(c_t)/dt}{u'(c_t)} = \Big[\rho + \boldsymbol{\mathcal{S}_t} \cdot \boldsymbol{\eta(x_t)}\Big] - r, \qquad \boldsymbol{\eta(x)} \equiv \textit{inc. elasticity of cons. } \frac{\partial \log c(x)}{\partial y}$$

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- Under rational expectations ($\mathcal{S} = 0$), standard Euler equation
- Sentiment distortions depend on distance to borrowing limit

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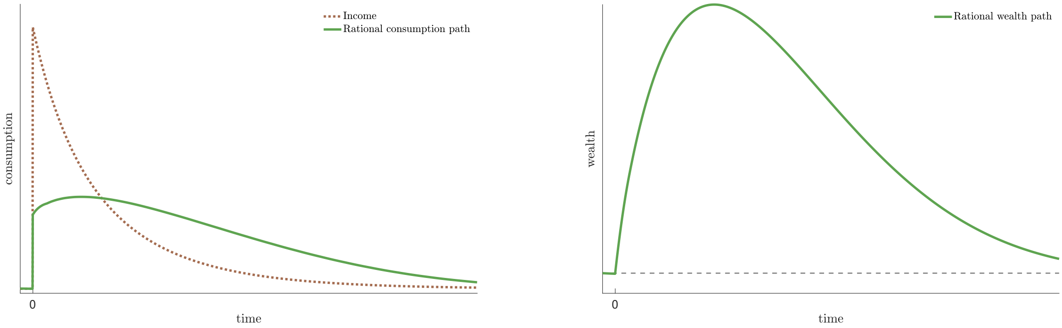

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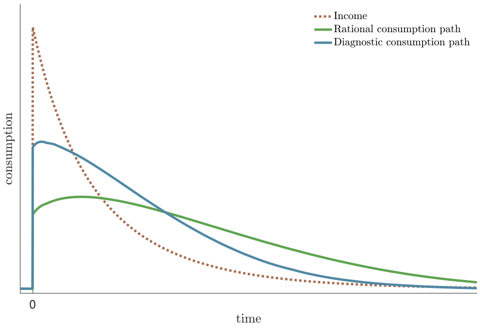


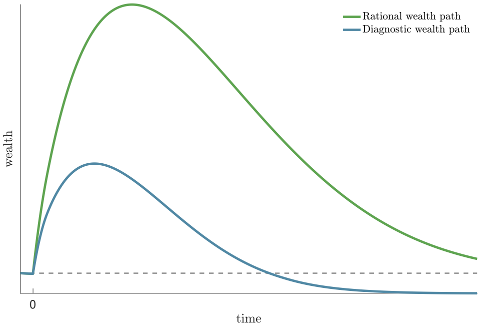

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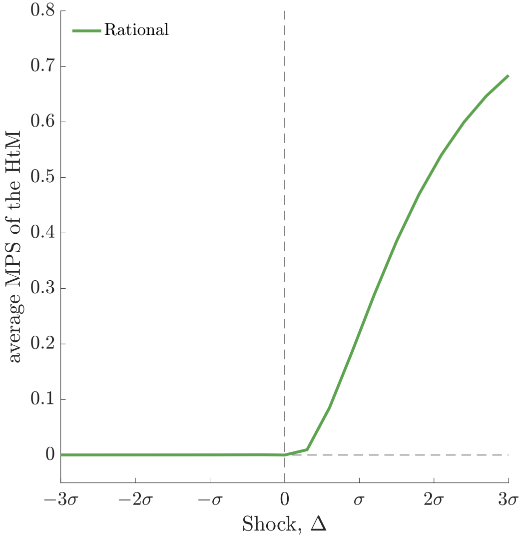

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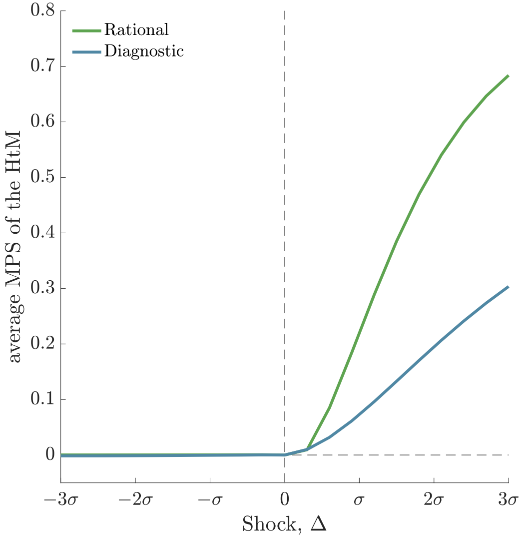
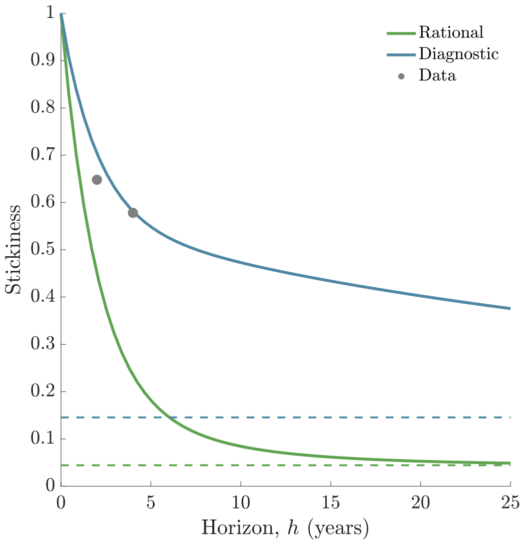


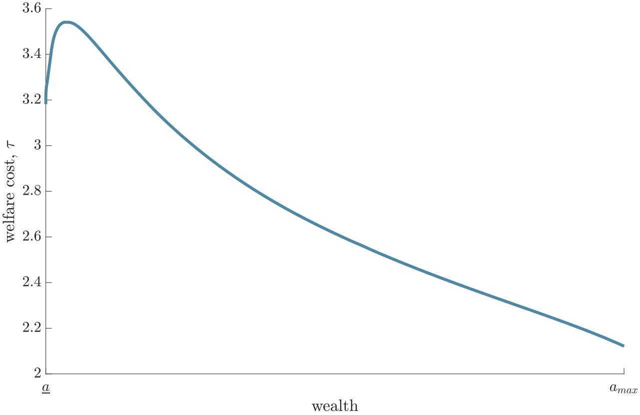
- Marginal propensity to save (MPS):
  - *Average share of an income shock saved by a household over a period* $\tau$
- Stickiness of the HtM:
  - *Average probability of being HtM in period* $h$ *conditional on being HtM in period* $0$



# DISTRIBUTED WELFARE COST

- Welfare cost: consumption tax $\tau(a, y)$ equating expected welfare:

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \log \left[ (1 - \tau(a_0, y_0)) c^{RE}(a_t, y_t) \right] dt = \mathbb{E}_0 \int_0^\infty e^{-\rho t} \log \left[ c^{DE}(a_t, y_t, \mathcal{S}_t) \right] dt \quad \Bigg| \begin{array}{l} a_0 = a \\ y_0 = y \\ \mathcal{S}_0 = 0 \end{array}$$



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